PHYSICS 206a
HOMEWORK #7
SOLUTIONS

1. **A teaspoon of sugar contains 62790 Joules of chemical potential energy. If a building is 3 meters per floor, how many floors up would a 75 kg person need to climb to use up the energy in a teaspoon of sugar? (An ungraded variation for your consideration: A can of Coke has about ten teaspoons of sugar. How high does that person need to climb to "burn off" a can of Coke?)**

This is just a very simple calculation, albeit a very disturbing one: The total work done by a person climbing a building is. This is the force the person exerts against gravity (*mg*) times the height they climb. It is equivalent (since gravity is a conservative force) to the change in gravitational potential energy of the person. (We neglect the K.E. of the person—let's assume they maintain the same speed going up that they had walking horizontally into the building.) Setting this equal to the energy available as chemical potential energy in a teaspoon of sugar and inserting the mass of the person and the acceleration due to gravity, we have $W = mgh = 75\,\text{kg} \times 9.8\,\frac{\text{meters}}{\text{second}^2} \times h = 62790\,\text{Joules}$. This can easily be solved for h to yield $h = 85.4\,\text{meters}$. And, since we have 3 meters per floor, we just divide by three to find that the total number of floors is slightly over 28. Scary! That can of Coke would fuel nearly three trips up the Empire State Building! (And let's not even think about what a Krispy Kreme doughnut would be good for.)

Notice that, in this case, we've transformed one type of potential energy (chemical) to another type of potential energy (gravitational). Students new to these concepts frequently make the erroneous assumption that one can only transform potential energy into kinetic energy. This isn't true: Any form of energy can, in principle, be transformed into any other kind of energy.

This problem points out the fact that human beings are chemical engines. We don't convert chemical potential energy directly to mechanical work. There are essential inefficiencies in the way we operate. One must be careful in considering how much energy a specific system needs: It needs at least enough energy to do the mechanical work $W = \vec{F} \cdot \vec{d}$. But it may well need significantly **more** energy to perform tasks "under the hood" that are never seen externally.

5. **For each of the situations described below, state whether the forces involved are conservative or dissipative:**
    a. **A baseball bat swings and hits a ball.**
    b. **An outfielder catches the ball.**
    c. **A man places a bowling ball on a tall shelf.**
    d. **The bowling ball falls down and shatters the floor below it.**
    e. **A water molecule is ripped apart into its constituents—hydrogen and oxygen.**

I wanted to get you thinking, in this set of problems. In each of these cases, there are several different answers that might be given. Let's walk through a few of them. Recall that conservative forces lead to work that becomes *either* kinetic energy (K.E.) or potential energy (P.E.). P.E. is energy representing work done that is still available to the system—getting at it can be a trick, in many cases, but it's still there. Dissipative forces, on the other hand, lead to work that is lost to the system. Certain keywords that give a clue that a force is dissipative are: Rip, break, stick, rub, and bend. ("Bend" can go either way. If something bends "elastically," i.e., if it bends and can spring back, then the force is conservative. If it bends and stays bent, the force is dissipative.) Let's analyze the situations listed above:

a. The baseball bat swinging has a lot of K.E. The ball also has a lot of K.E. When the bat hits the ball, the first thing that happens is that the ball deforms. This robs the ball of all of its K.E. and it robs the bat of some of its K.E. But a lot of that energy goes into an elastic deformation of the ball (and the bat—bats bend and compress during the violent collision between bat and ball). Here's the neat part: If the bat is swinging fast enough, the bat accelerates the ball up to a very high speed while the ball is still compressed. (This is why it's desirable for balls to stay in contact with bats for as long as possible, which, in turn, is why unscrupulous baseball players coat their bats with sticky stuff, like pine tar.) After a little while, the ball un-compresses, regaining much of the K.E. (that had originally been in both the bat and the ball) that had been stored as P.E. in the ball's deformation. So the ball gets back much of its original K.E., some of the bat's original K.E., *and* some K.E. due to work done on the ball by the bat (and, ultimately, by the guy *swinging* the bat). So this situation is *primarily* conservative.

   That said, there are still dissipative factors involved in this situation. The wrapper on a baseball gets torn. Bats get dented and get micro-cracks in them (which, ultimately, leads to them shattering). One can, literally, hear some of the dissipative forces: Even many meters away from a bat hitting a ball, the sound is quite loud. Thousands of tons of air were moved by the sound wave that allows us to hear the impact! So dissipative forces abound in this situation, even though the desired behavior consists of the conservative forces.

b. When the outfielder catches the ball, quite a different situation exists. The ball has a huge kinetic energy when it reaches him. (Often, this was temporarily transformed to gravitational potential energy along the way, but that's all gone when the ball descends into the outfielder's hand.) But he catches the ball and *all* of that hard-won K.E. goes away. Can it be gotten back? Nope. It's gone. This involves essentially purely dissipative forces.

c. To place a bowling ball on a tall shelf, the man had to do a fair amount of work. But this was almost purely in the lifting of the ball against the force of gravity. So the ball gains P.E. This is still available—all we need to do to get it back is to let the ball fall down. Now, it is true that there really isn't a good conduit for the man to get the energy back. If the ball gains K.E. by falling off a shelf, the man can't eat it to regain the chemical P.E. he used doing the work needed to get the ball up there in the first place. But the definition of P.E. doesn't demand that the energy be regained in any particular form. As long as it's still available to the system to do any kind of work, the energy is potential.

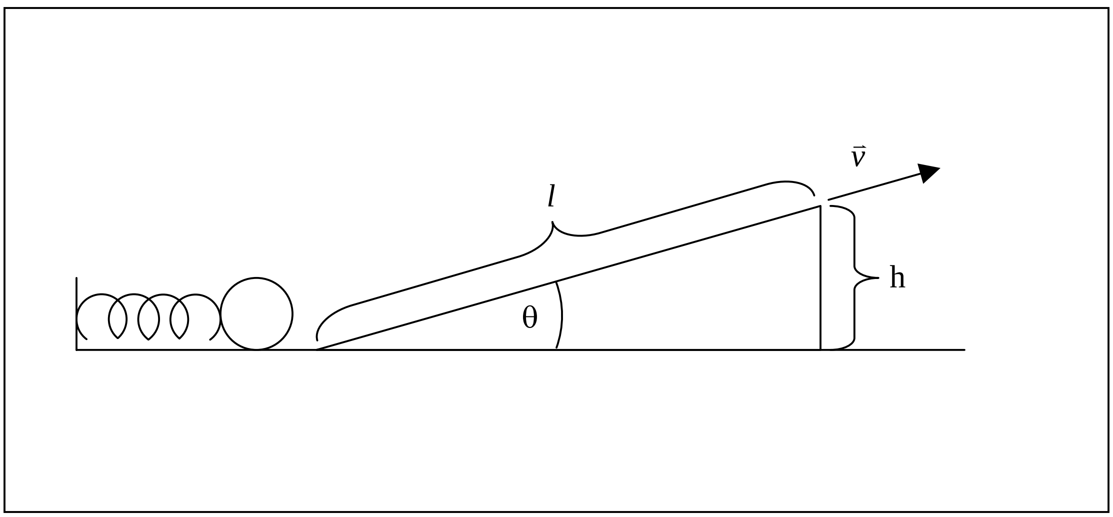
d. In this case, there are two phases: The falling and the smashing. When the ball falls off the shelf, the P.E. that the ball gained while being placed there is transformed into K.E. via gravity. This is a conservative force. But then it hits the ground. The floor is smashed. It takes work to break things like floorboards. That work cannot be gotten back. So the force which breaks the floor is dissipative.

e. The force that holds the atoms in a molecule together is primarily the "electrostatic force." This is a conservative force. Molecules can have either positive or negative P.E. A molecule which has a negative P.E. requires work to break it apart: This is the sort of reaction that Chemists call "endothermic." The P.E. of the molecule is *less* than the P.E. of the constituent atoms when they are far apart. A molecule which *releases* energy when it is broken apart has a positive P.E. This is the sort of reaction that Chemists call "exothermic." The constituent atoms have less P.E. when they are far apart than the molecule had. Either way, the force involved is conservative.

You'd be right to be a bit confused by this last result: I told you previously that when something breaks the force is dissipative. Here, a molecule is breaking, isn't it? Well, yeah. But not in the same sense as a piece of wood cracking. There are some amazing subtleties to this that we will discuss later this semester, but, for now at least, the big difference is simply between "microscopic" and "macroscopic." On the atomic level, *all* forces are conservative. Only when systems get complex do dissipative forces come into play.

6. **The potential energy of a compressed spring is given by $P.E. = \frac{1}{2}kx^2$ (we'll study springs in more detail later this semester) where k is a constant dependent on the particulars of the spring and x is the amount by which it is compressed. Consider the following situation: A spring, oriented horizontally, with $k = 70\frac{Newtons}{meter}$ is compressed by 13 cm. It is then allowed to expand, pushing against a ball with a mass of 37 grams. The ball rolls up a ramp with an incline of 17° relative to the horizontal. The ramp is 3 meters long.**
    a. **What will the ball's velocity be when it leaves the ramp?**
    b. **What will the ball's *speed* be when it hits the ground?**

Oh, dear! This does look complicated, doesn't it? Well, it's actually not too bad, if you have a good understanding of Conservation of Energy. Let's begin with a little picture:



Now, think about what's going on: Someone does some work to compress the spring. The spring stores that work in the form of its own P.E. until it is released. It then transfers that P.E. to the ball, which starts to roll with a certain amount of K.E. (In fact, a certain amount of energy goes into "rotational kinetic energy" which we haven't yet studied. Perhaps I'll re-assign this problem after we've studied that. It actually does have a profound impact on the answer you'll get.) But we don't need that information just yet. This is because the ball goes up the ramp. This converts a certain fraction of the K.E. into gravitational P.E. But not all of it. The ball still has some K.E. when it gets to the top of the ramp. So it flies off the end of the ramp with a speed v. Before we go too much further, let's do some calculations.

The total energy of the system is just whatever initial potential energy was stored by the spring. This is the constant in our conservation of energy equation.

I.e., $P.E. + K.E. = constant = \frac{1}{2}kx^2$. In order to find the speed at the top of the ramp, we'll need the kinetic energy, so we solve for that:

$$K.E. = \frac{1}{2}mv^2 = \frac{1}{2}kx^2 - P.E.$$

To go on, we'll need to figure out the P.E. At the top of the ramp, the only P.E. is that due to gravity and we have a simple equation for this: $P.E._{gravity} = mgh$. Be careful using this: h is the distance from the place where we choose P.E. to be zero. In this case, it makes a lot of sense to pick the ground as that location. So h is the height above ground. It won't always be!

Now, from the picture, we can see that the height above ground of the ball can be simply found to be $h = l\sin(\theta)$ where l is the distance the ball travels along the ramp. So $P.E. = mgl\sin(\theta)$.

We can directly substitute this into our energy conservation equation to get $K.E. = \frac{1}{2}mv^2 = \frac{1}{2}kx^2 - P.E. = \frac{1}{2}kx^2 - mgl\sin(\theta)$. A bit of algebra solves for the speed: $v = \sqrt{\frac{kx^2 - 2mgl\sin(\theta)}{m}}$. (Note my use of symbols throughout: Break yourselves of the insidious habit of substituting numbers into your solutions before the very end! Symbols convey far more meaning than numbers and numbers easily hide errors from you, preventing you from fixing them, but expose them to critics [like me and your grader]!)

We're not done yet! The question asked for the *velocity* of the ball at the instant it leaves the ramp. We have its speed, but we lack the directions. When the ball leaves the ramp, it initially has exactly the speed it had when it was on the ramp and it initially travels in exactly the same direction as the surface of the ramp. (Of course, this begins changing immediately, but at the *instant* the ball leaves the ramp, this is the case.) It would be minimally acceptable simply to express the velocity, therefore, as $\vec{v} = v$ "17° relative to the horizontal." But using components is far preferable. In fact, very soon now, I'm going to *insist* that vectors be represented only in component form—you've graduated from that kiddie stuff! Doing it this way is quite simple. Using the methods that, by now, are very familiar to you, we get $\vec{v} = \sqrt{\frac{kx^2 - 2mgl\sin(\theta)}{m}}\left[\cos(17°)\hat{x} + \sin(17°)\hat{y}\right]$. *Now* you can substitute numbers into the equation, if you want.

4. **Which has a bigger change in kinetic energy: A ball thrown against the wall which bounces back to the thrower or one which (due to a nasty practical joke) sticks to the wall?**

Let's call the (kinetic) energy of the ball just before it hits the wall E. If the ball sticks, then its energy after the collision is zero. If the ball bounces off at any speed at all, its kinetic energy is going to be greater than zero so the change in energy is going to be:

$$\Delta K.E. = Energy\ before\ hitting - Energy\ after\ hitting$$

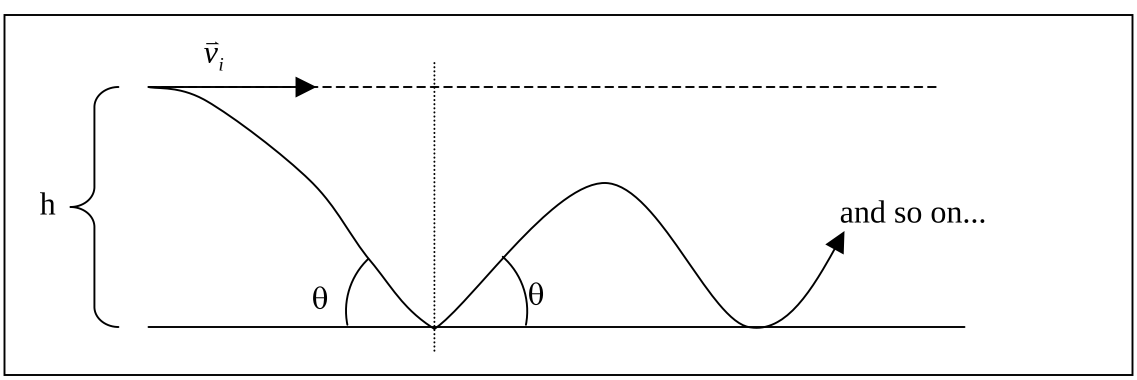
$$= E - Something\ greater\ than\ zero$$

There is no way the wall is going to add energy to the ball. (If it were a baseball bat, golf club, or tennis racquet things would be different. In those cases, *K.E.* is transferred to the ball from something into which a human being is continuously adding energy.) So the energy after hitting is *at most* the same as the energy before hitting. For the case in which the ball bounced, the change in K.E. might be anything between zero and E, but always less than E. So the ball which sticks always has the bigger change in energy.

5. **A rubber ball with a mass of 210 grams is thrown exactly horizontally from a height of 2 meters with an initial speed of $v = 13\frac{m}{s}$. When it strikes the ground, it bounces off at the same angle it made when it landed. During the bounce, it loses 20% of its kinetic energy to dissipative forces. How high does it go after bouncing?**

A major reason I assigned this problem is to continue the process of breaking you of a very bad habit of thought that has been cultivated in many of you by the sorts of problems you've been assigned in the past: It is my conjecture that you believe (unconsciously) that all problems can be solved in a single step. When I state it this way, you probably say "well, I don't believe that at all!" Whether the conjecture is true or not, I'm still certain that practice in solving multi-step problems is a good thing.

Here we have a problem that simply *cannot* be solved in a single step. It's best attacked as two, distinct problems. The situation appears in the picture below:



(Sorry for the nasty artwork! Let me know if it really doesn't make sense.)

What we have here is *two* problems which can be solved using conservation of energy, but the constant (the total energy) changes from one to the next, so we need to deal with them one at a time. I've drawn a (vertical) dotted line to indicate where the first problem is separated from the second.

Beginning on the left: This is a class case of conservation of energy. We have the standard equation we begin with in all such cases $P.E + K.E. = constant$. We just need to figure out what the constant is. Of course, the constant is the total energy, so let's figure out what that is. Let's pick the zero point for potential energy to be on the ground. This means that the ball starts with some gravitational *P.E.* It also (as we are told in the statement of the problem) has some *K.E.* as well. Let's give the constant a name—let's call it K, for convenience. So we have $P.E + K.E. = K = mgh + \frac{1}{2}mv_i^2$. (Note that I've called the initial speed, v_i where $v_i = 13\frac{m}{s}$.)

Since the system is conservative, until the bounce, this will be the total energy when the ball is just about to hit the ground, as well. At that instant, just before the ball hits the ground, the potential energy is zero (we picked the ground to be the place where *P.E.* is zero). So we can say $\frac{1}{2}mv_1^2 = K$.

Why did I bother finding this? Well, we were told that the angle after the bounce is the same as the angle before the bounce. But we weren't told what that angle is! I'd sure like to know—it will come in very handy in a little while. So I'd like to find the magnitude of the velocity, which I've called v_1. Solving the above equation, we get $v_1^2 = 2gh + v_i^2$. (If that doesn't look familiar, look at it again: This is our old friend $v_f^2 - v_i^2 = 2ad$ in a slightly modified form.) We *also* know the *x* component of the velocity: Since the only acceleration in this problem (so far) was that due to gravity, the *x* component of the velocity has been left unchanged from its initial value. So we know the *x* component of velocity and the magnitude of the velocity as well. This means that we can find the cosine of the angle and, with one last step, the angle itself: $\cos(\theta) = \frac{v_{1x}}{v_1} = \frac{v_i}{v_1}$. I'll just leave it like that, for now. Next, we bounce.

The ball hits the ground. A bunch of complicated stuff happens, but this takes only a small amount of time. At the end of this time, the ball has finished its bounce but it hasn't gone anywhere yet. So its *P.E.* is still zero. All of its energy is *K.E.*, but the total energy is reduced from what we found previously. Now we have $\frac{1}{2}mv_2^2 = 0.8 \times K$. Where did the 0.8 come from? Well, it lost 20% of the energy that it had, so it's still got 80% of it's original total energy. Thus,

$\frac{1}{2}mv_2^2 = 0.8 \times K = 0.8 \times \frac{1}{2}mv_1^2$. A bit of algebra gives $v_2^2 = 0.8 \times v_1^2 = 0.8 \times \left(2gh + v_i^2\right)$. Or, finally, $v_2 = \sqrt{0.8 \times \left(2gh + v_i^2\right)}$.

We want the total height after the bounce. So we'll need the *y* component of the velocity. From the theorem of Pythagoras, $v_2^2 = v_{2x}^2 + v_{2y}^2$. So $v_{2y}^2 = v_2^2 - v_{2x}^2 = v_2^2 - v_2^2 \cos^2(\theta)$. We can now use $\cos(\theta) = \frac{v_i}{v_1}$ and $v_1^2 = 2gh + v_i^2$ to get $v_{2y}^2 = v_2^2 - v_2^2 \cos^2(\theta) = v_2^2 - v_2^2 \times \left(\frac{v_i^2}{2gh + v_i^2}\right)$. And, finally, substituting $v_2 = \sqrt{0.8 \times \left(2gh + v_i^2\right)}$, we have

$$\begin{aligned} v_{2y}^2 &= v_2^2 - v_2^2 \times \left(\frac{v_i^2}{2gh + v_i^2}\right) \\ &= 0.8 \times \left(2gh + v_i^2\right) - 0.8 \times \left(2gh + v_i^2\right) \times \left(\frac{v_i^2}{2gh + v_i^2}\right) = 0.8 \times 2gh \end{aligned}$$

Of course, we can now use $v_f^2 - v_i^2 = 2ad$ to conclude that $2gh_2 = v_{2y}^2 = 0.8 \times 2gh$ so $h_2 = 0.8h.$ Using numbers, $h_2 = 1.6m$.

Confession time: I gave many of you the wrong answer on this when you talked to me about it! Many people came up with this answer, but, due to an algebra gaff on my part, I told them that they were wrong. Mea maxima culpa! I apologize profusely. I was wrong, you were right.

Frankly, this is a very surprising result! Note that it is only true because the angle of the bounce is the same as the angle of the impact. In an inelastic bounce, this is *not* always the case! This is why my intuition failed me: I know that the general answer is more complicated. But, by giving you a simple geometry, we got a very elegant answer. One of the many things that I love about my job is that I get to learn little tidbits like this from time to time. Thank you for the opportunity!

6. **Consider again the situation given in Problem #9 of Assignment #6: A man drags a heavy box across the floor at a constant speed using a rope. The rope makes an angle θ with the floor. The man exerts a force F_T on the rope. Assume the box has a mass of 15 kg, θ=17°, F_T=75 N. If the man walks at a constant speed of 0**$.7 \frac{\text{meters}}{\text{second}}$**, how much power does he expend? (Note: This only makes sense if there is friction in the problem, so assume that there is friction—it won't affect your answer in any way and you will not need an explicit value for the coefficient of friction. But the statement of the problem is contradictory without it. Can you see why?)**

All we need to do is remember that power is simply energy divided by the time over which that energy is produced or used. To see the solution to this problem, imagine that the man walks a distance *d*. How much work does he do? Recalling that work is force times the distance traveled in the direction of the force, from the previous assignment, we have $W = \mathrm{F_T} d\cos(\theta)$. To find the power produced (or used) we just divide this by the time it took him to walk that distance $P = \frac{\mathrm{F_T} d\cos(\theta)}{t}$. But the distance traveled divided by the time it took to travel that distance, $\frac{d}{t}$, is just the definition of speed. So $P = \frac{\mathrm{F_T} d\cos(\theta)}{t} = \mathrm{F_T} v\cos(\theta)$. Stated in terms of the "dot" product, $P = \vec{F} \cdot \vec{v}$.

7. **Which has a bigger change in momentum: A ball thrown against a wall which bounces back to the thrower or one which (due to a nasty practical joke) sticks to the wall?**

This problem was intended to drive home the importance of the fact that momentum is a **vector** quantity. Let's take the initial momentum of the ball to be $\vec{p} = p_0 \hat{x}$. For convenience, let's just ignore the fact that the ball arcs and will have some *y* component to its velocity and momentum due to gravity. This makes no difference in the problem, but it does make for extra work to solve it. Now, when the ball sticks, the momentum at the end of the trip is zero—the ball stops moving. So $\Delta\vec{p} = \vec{p}_{end} - \vec{p}_{begin} = 0 - p_0\hat{x} = -p_0\hat{x}$.

On the other hand, if the ball bounces back with some speed, its return momentum will be $\vec{p}_{end} = -\vec{p}_1\hat{x}$. Here's the crucial thing: Notice the sign on $\vec{p}_{end}$. If the ball's momentum while traveling *to* the wall was positive, then its momentum traveling *away* from the wall must be negative! It's size is not important in this problem, only its direction. When we calculate the change in momentum in this case we get $\Delta\vec{p} = \vec{p}_{end} - \vec{p}_{begin} = -\vec{p}_1\hat{x} - p_0\hat{x} = -(p_1 + p_0)\hat{x}$. Since we're assuming that p_1 is greater than zero (after all, it can't be less than zero and if it's *equal* to zero, then the ball stuck, which we're assuming isn't the case), $p_1 + p_0$ must be greater than p_0. Thus, the change in momentum is *bigger* for the ball that bounces.

In the special case that the ball bounces back with the same speed as that with which it was thrown, the change in momentum is *twice* the original momentum! Compare this with the result you found for energy in the previous assignment.

8. **With what minimum speed would a housefly (with a mass of 1 gram) have to be thrown against a Volkswagen (with a mass of 1300 kilograms) traveling at $22\,\frac{\text{meters}}{\text{second}}$ in order to get it to stop?**

After the impact of the fly against the VW, the speed of both of the bugs is zero (that's what stopping is, after all). Since their speeds are both zero, the total momentum of the system (VW + fly) is zero. Since momentum is always conserved, if the total momentum of the system is zero *after* the collision, it must be zero *before* the collision as well. Thus we can say $\vec{p}_{total} = \vec{p}_{fly} + \vec{p}_{VW} = 0$. Since this is clearly a one-dimensional problem, we can rewrite this without the vector symbols, which gives: $p_{total} = p_{fly} + p_{VW} = 0$. This means $p_{fly} = -p_{VW}$. Writing this out explicitly in terms of mass and speed, we have $m_{fly} v_{fly} = -m_{VW} v_{vw}$ which can be solved for the speed of the fly, $v_{fly} = -\frac{m_{VW}}{m_{fly}} v_{vw}$. Inserting numbers gives $v_{fly} = -\frac{1300\,\text{kg}}{1\times 10^{-3}\,\text{kg}} 22\,\frac{\text{meters}}{\text{second}} = -2.9\times 10^{7}\,\frac{\text{meters}}{\text{second}}$. The minus sign indicates, as we knew, that the fly is traveling in a direction opposite to that of the car. Note that this speed is 10% of the speed of light—the fastest speed that it is possible for anything to have. At this speed, we'd have to use Einstein's theory of relativity to get a precise answer! So you don't have to worry about an errant bug causing you to suddenly stop on the freeway.

9. **A molecule of carbon monoxide (CO), which consists of one atom of carbon and one atom of oxygen, is forced to break apart by the addition of a certain amount of energy. There is a net excess of energy of 3×10^{-19} joules which all goes to kinetic energy of the "fragments" (i.e., the atoms). What are the velocities of the two fragments? (Carbon has a mass of 12 amu and oxygen has a mass of 16 amu.)**

It may not be obvious to you, but this problem is an inelastic collision. Granted, the particles (the atoms, in this case) are flying apart rather than coming together, but all that means is that the clock is running backwards. Imagine I'd told you the atoms strike each other and stick together and that they have 3×10^{-19} Joules more energy before they strike than after they stick. See, now *that's* an inelastic collision! Same thing. This is a powerful insight and if you can see it in this way it is a sign that you understand what's going on at a sophisticated level.

We have two unknown quantities here: The speed of the carbon atom and the speed of the oxygen atom. (Because there are only the two particles, this must be a one-dimensional problem, thus we can ignore the vector aspects and work with speed and not velocity.) We know that we will, therefore, need two algebraic equations.

A powerful problem-solving strategy is to begin by listing the things we absolutely know about the system. It's crucial that we get this list correct: Be certain that you really know the items on it are true and that you're not being driven by wishful thinking! (E.g., a lot of people simply assume that the two speeds are the same. Untrue!) That said, I can think of two things that I know are true: (1)The momentum of the system is conserved with a total value of zero (we are safe in assuming that the initial velocity of the molecule is zero—even if it isn't, we know that we can change our reference frame to one in which this is the case, so we might as well keep things simple), and (2)the sum of the *K.E.s* of the atoms will be 3×10^{-19}**.** The first I know because it is always true (that is, the momentum is always conserved—it is not always conserved at a value of zero, however; this needs to be determined for each problem). The second I know because it was stated in the problem itself. Writing these out as equations, we have

$$m_c v_c = -m_o v_o \quad \text{and} \quad \tfrac{1}{2} m_c v_c^2 + \tfrac{1}{2} m_o v_o^2 = 3\times10^{-19}\ \text{Joules}.$$

Let's solve the first of these to get v_c in terms of v_o. This gives $v_c = -\dfrac{m_o}{m_c} v_o$. This can be substituted into the *K.E.* equation to give $\tfrac{1}{2} m_c \left(\dfrac{m_o}{m_c} v_o\right)^2 + \tfrac{1}{2} m_o v_o^2 = 3\times10^{-19}\ \text{Joules}$. Multiply both sides by 2 and combine terms on the left to get $\left(\dfrac{m_o^2}{m_c} + m_o\right) v_o^2 = 6\times10^{-19}\ \text{Joules}$ which gives $v_o = \sqrt{\dfrac{6\times10^{-19}\ \text{Joules}}{\dfrac{m_o^2}{m_c} + m_o}}$. So far this was all pretty mechanical. Here's the first real trap. First of all, for those of you who obsess on putting numbers in as soon as humanly possible, you've really made a lot of work for yourself! Now is the first place where numbers really should be substituted (even expressing the *K.E.* as a number wasn't the prettiest thing in the world—it would have been better simply to keep calling it *K.E.*). Second, I gave you the masses in amu (Atomic Mass Units). Don't convert these to kg just yet! Wait until you have to. For now, working in amus, which are small integers, makes a lot more sense. But we must remember to convert from amus to kg before we're done.

Substituting the masses of carbon and oxygen into the above equation gives $v_o = \sqrt{\dfrac{6\times 10^{-19}\text{ Joules}}{\dfrac{m_o^2}{m_c}+m_o}} = \sqrt{\dfrac{6\times 10^{-19}\text{ Joules}}{\dfrac{16^2\text{ amu}^2}{12\,\text{amu}}+16\,\text{amu}}} = \sqrt{\dfrac{6\times 10^{-19}\text{ Joules}}{37.3\,\text{amu}}}$. *Now* convert the amu to kg. There are 1.67×10^{-27} kg in each amu, so

$$v_o = \sqrt{\frac{6\times 10^{-19}\text{ Joules}}{37.3\,\text{amu}}} = \sqrt{\frac{6\times 10^{-19}\text{ Joules}}{37.3\,\text{amu}\times 1.67\times 10^{-27}\dfrac{\text{kg}}{\text{amu}}}}$$

$$= \sqrt{\frac{6\times 10^{-19}\text{ Joules}}{6.23\times 10^{-26}\text{ kg}}} = 3.1\times 10^{3}\,\frac{\text{meters}}{\text{second}}$$

To find v_c, we just use this value along with the masses in amus (no reason to convert at all in this step since the masses occur in a ratio—their units just cancel). This gives $v_c = -\dfrac{m_o}{m_c}v_o = -\dfrac{16}{12}\times 3.1\times 10^{3}\,\dfrac{\text{meters}}{\text{second}} = -4.1\times 10^{3}\,\dfrac{\text{meters}}{\text{second}}$.