PHYSICS 206a
HOMEWORK #5
SOLUTIONS

"Foxtrot" by Bill Amend, February 10, 2006

Before beginning with the solution set, I wanted to discuss some flaws in the approach to problem solving I have seen on the part of students over the years that are certainly appearing in the current "crop" of 206a students. I hope you will take the following advice seriously—it goes to the very heart of what we are doing in this course.

To begin with, many students approach their homework assignments in the same spirit as the young lady pictured above. They view the goal of the assignment to be "write the correct numbers down on the piece of paper." Essentially, the assignment is thought to be a mini-exam. As stated in the Course Description, this is not the case and you are setting yourself up for a disaster if you treat it that way!

I structure the questions on the assignments specifically to make you think about particular concepts in Physics with which I know you will have difficulty. The problems are placed there to force you to think through those difficulties. Resorting to a solution method other than your own thoughts is like going to the gym and using a motor to lift weights for you: The goal of lifting the weights isn't to get them up into the air, it's to give you the exercise inherent in the lifting of them. As with someone working out with weights, you may need to be "spotted" to avoid simply dropping the weight. That's what office hours with me or group work with your colleagues are for. It's not a replacement for doing the work. It's a last little boost to get you over the top.

So let's assume that you *are* sitting yourself down and really giving the problems a shot. How are you approaching their solutions? One stunningly common phrase that I and my colleagues hear from introductory level students is "I can't find the formula to use!" or some variation on that. We universally wince at this. We do so because it indicates that you've gone wrong before you even began. Here's the key: ***There aren't any formulas!*** O.K., get up off the floor. It will be alright. Let me explain that statement. When I (or anyone else sophisticated in the use of Mathematics) writes down an equation, for example $a+b=c$, we don't see it as writing down a "formula for *c*." What we're doing is writing down a *relationship* between *a*, *b*, and *c*. This is a sentence that tells how the various elements interact with each other. By analogy, let's say I write a sentence in English, "the girl has red hair." I have not just written down a definition of "girl." I've written a relationship between a girl, her hair, and a color. There are several ideas expressed in this sentence. Depending on what I want to know, I can pull out different aspects by looking at the sentence in different ways.

When we derive a mathematical expression for a concept in Physics, do not think of it as a formula for getting a single "output" if a particular set of "inputs" are presented—that way lies failure! Rather, recognize that the expression is a statement of an idea. It is an idea that relates the various components of the expression to each other. Is $\vec{r} = \frac{1}{2}\vec{a}t^2 + \vec{v}_0 t + \vec{r}_0$ a formula for the position of an object? Well, if you think of it that way, then when you're given a distance and asked to find a time you'll be clueless. When you're given a distance and a time and are asked to find an acceleration, you'll be floored. And when you're given a time, a distance, and an acceleration and are asked to find an initial velocity you'll simply be crushed. Think of this as the *relationship* between the various aspects of the motion of an object and time and you will have the mental agility to adapt it to various applications. It is not an exaggeration to say that one of the primary goals of this course is to train you to think about mathematical expressions this way so your level of success in this course will depend on how well you manage to adapt to this way of thinking.

But your solution to the problems shouldn't begin with "the formula" anyway. You must begin each problem by thinking about it. That sounds pretty obvious, doesn't it? But how much time have you been spending before looking for something with the right variables in it? Be honest! My guess is that it's almost none for many of you. When you read a problem, visualize what's happening in it. (Drawing a simple picture at this point is invaluable!) Then interrogate yourself: Why do you think the system will behave the way you're visualizing it? Can you pick out the Physical principles you're invoking in your mental picture? One prime example of how this sort of mental interrogation can be useful appears in problem #8 of the last assignment—the one about the bomb dropped out of the airplane. If I asked you what the speed (relative to the ground) of the bomb is *before* being released from the plane, virtually all of you would have answered, correctly, that it is the same as that of the plane. However, when asked what the speed of the bomb is (again, relative to the ground) immediately *after* leaving the plane, some of the students I spoke with responded that it is zero. If you get in the habit of walking through your thought process and critically evaluating it, you will catch mistakes like this before they get you into too much trouble: The bomb's velocity cannot change unless a net force acts on it. By Newton's first law, it will continue traveling in a straight line at a constant speed without such a force. Gravity will accelerate the bomb downward, but the horizontal component of its velocity will be unchanged since there is no force in the problem acting horizontally. If you were in the habit of visualizing situations before leaping in with a formula, you'd have recognized that the bomb's speed certainly can't jump from something to nothing simply because it's no longer in the airplane!

Having visualized a problem at the beginning also helps you to train yourself to view the world correctly: Occasionally, you will make an error in what you expect a system to do even if you've diligently visualized it. When your ultimate solution is finalized, you can go back and compare it to your initial prediction. Are they consistent? If not, either you made a mistake in your problem solving or in your visualization. In the former case, catching the discrepancy will help you to find your blunder. In the latter case, you should take the opportunity to revise your view of the world. This is a rare and wonderful opportunity which you should take advantage of! It's not every day that one gets a chance to change how one sees the universe!

So, you've now visualized the problem. You have a good sense of how things are going to evolve within the situation you've been handed. Now what? Now, make an inventory of what you *know*. Make sure that you really do know these things and are not just making assumptions. Also, make sure that you know what you're looking for. Reread the question a few times. I can't tell you how many times I've handed out a low grade to a student who simply didn't answer the question that was asked! Having laid out your "knowns" and confirmed your ultimate goals for the exercise, think about what physical principles might help you. You only have a handful to choose from, so working through the list won't take so long: Is energy conserved? Are there net forces involved? (Make sure that you have a good idea of what you expect to matter and what you think can be ignored—this list may need to be modified as you solve the problem.) *Now* is when you can go to your toolbox of mathematical statements. The equations in that toolbox will provide you with guidance for the next step.

One last thing: Our educational system rewards and reinforces the notion that problems can be solved in a single step. This is not true! The vast majority of problems require multiple steps—sometimes many steps. If you go into the problem looking for the one magic key that will unlock the door, you will likely wind up stranded. Seeing that a problem will take multiple steps and seeing, ahead of time, what those steps will probably be is an art that cannot be taught directly. Only practice will give you the experience needed to learn it. I have been working hard at this craft for over a quarter of a century and the worked solutions I give you should be viewed as ideals (although I certainly do still make mistakes from time to time!) to aspire to in your own solutions. Once you have worked a problem and given it your best shot, be sure to sit down with my solutions and spend some time and effort seeing where you and I did it differently. Learn from those differences! (If you discover that your method was more elegant than mine, by all means, tell me about it! I'm always eager to learn a new way of looking at the principles we study in this course.)

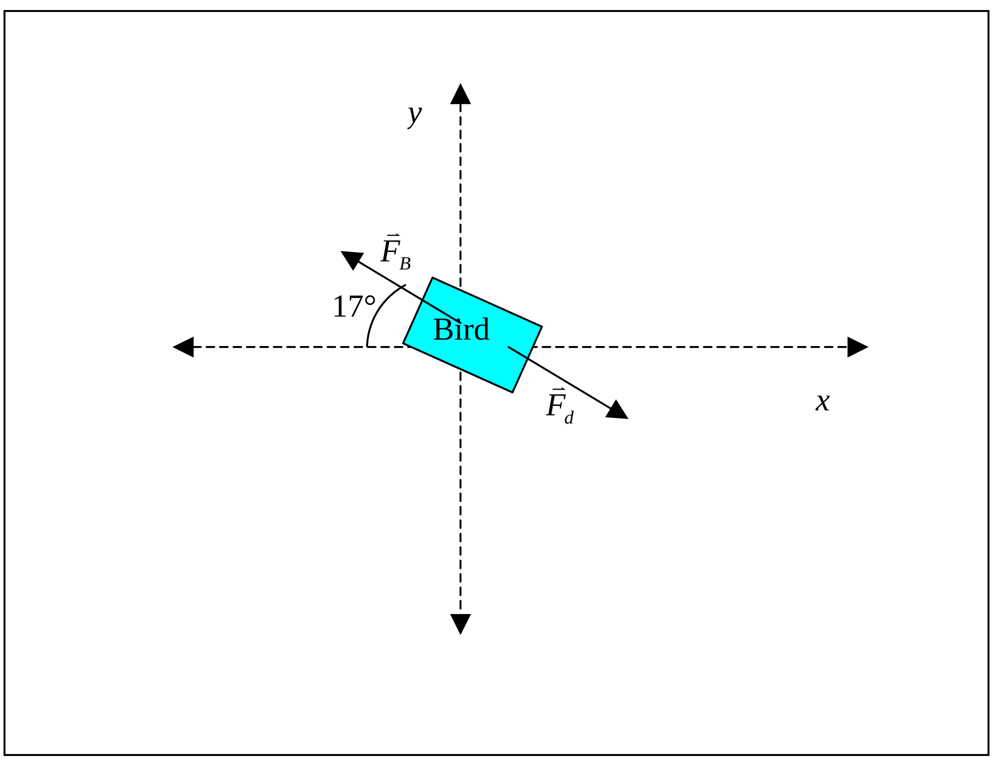
From there, the advice really becomes more specific, so I'll leave off for now. Everything stated above applies to *all* problems in Physics (and pretty much anything else as well, but I don't want to get too grandiose). For specific categories of problems, there are specific approaches that are useful: E.g., If it's got forces, draw free-body diagrams. If it's got vectors, be sure to decompose them into relevant components. Things like that. But getting started right is essential to getting to this point.

Above all, recall my words from the Course Description: This isn't a course about things. It is a course about thinking about things. You are here to learn how to think. You will never achieve that goal unless you actually push your thinking. Copying someone else's answers, being a passive observer of classroom discussions, or simply letting yourself fall into the "I don't know the answer" trap will not get you there! Approach the problems systematically and with your full mind working, be willing to be critical of your own thoughts, and recognize that each situation is different but that the same laws of Physics apply to all situations and your work will bear fruit.

1. **A bird flies at a constant velocity of $11\frac{m}{s}$ in a direction 17° North of West. The bird has a mss of $2.7kg$. The air exerts a drag force on the bird with a magnitude of $F_d = 5N$. Treating North as the $\hat{y}$ direction and East as the $\hat{x}$ direction**
    a. **Draw a free body diagram of the bird (since this is a *3-d* problem, it would be wise to draw two different free body diagrams: one a top view and one a side view).**
    b. **Give explicit values for all of the external forces acting on the bird. Express these in terms of the unit vectors.**

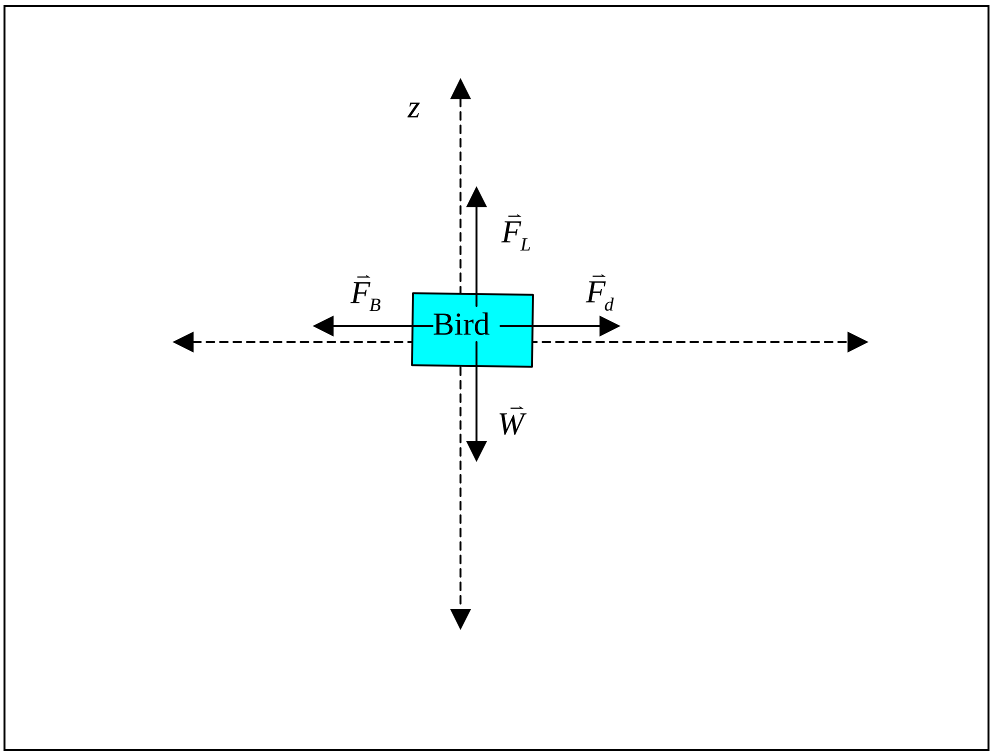
We begin, as we almost always do, with a little picture. In this case, the motion is perpendicular to the direction of gravity, so the "usual" coordinate system is perfectly fine for us: Up will be $\hat{z}$ and North and East will be $\hat{y}$ and $\hat{x}$, respectively, as stated in the problem. Since this is a 3-d problem, we'll have to do two free body diagrams to get the full picture. Let's do a top view first.

The drag force acts in a direction exactly opposite to the direction the bird is flying. Since we are told that the bird's velocity is constant, it must be exerting a forward force to counter this, so there will be two counter-acting forces that we will indicate in the free body diagram this is:



Next, we draw the side view. This one is just a smidge complicated. The reason that it's complicated is that our initial thought on how to look at this from the side is liable to be misleading. If we don't think about it, we'll draw the side view as though looking *along* the *y* axis. In the language of mathematics, this would make the sheet of paper the “*x-z* plane.” If we did this, we would have a real problem since the velocity vector of the bird would be pointed slightly into the page! I know how to draw that, but it’s rough. So we have to be a bit more thoughtful before we draw the side view.

Now, take a look at the picture above. Imagine a plane going into and out of the page that you’re looking at, but imagine that plane is parallel to the two vectors that I’ve already drawn. The velocity vector of the bird is parallel to the surface of that plane. So long as I am careful, I can make the side view drawing on that plane and not worry about the velocity vector pointing into the page after I’ve drawn it. This gives:



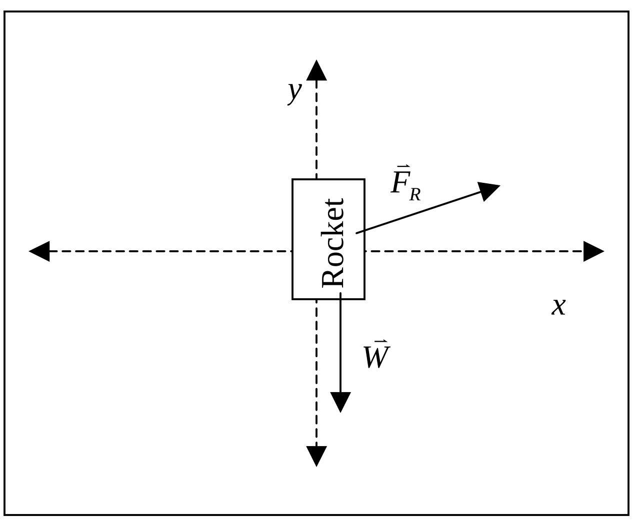
Now, finding the explicit values for the external forces is a piece of cake. We begin by realizing that the bird is flying at a constant velocity. That means it’s not accelerating. According to Newton’s Second Law, this means that the net force acting on it is zero. Since force is a vector, the only way for it to be zero is for each of its components to be zero. That is, the *x*, *y*, and *z* components must *each* add up to zero. Using this, we can deal with each force in turn.

We are given the drag force, $\vec{F}_d$. We are told that its magnitude is 5 Newtons. Since the drag force must be canceled by the forward force the bird exerts, which I have labeled $\vec{F}_B$, these two forces must be exactly the same size and must point in exactly opposite directions. So the size of $\vec{F}_B$ must be 5 Newtons and we can decompose it into its components since we know that it must point in exactly the same direction as the bird’s velocity. Using our basic trigonometric relations, we can say $\vec{F}_B = -5N\cos(17°)\hat{x} + 5N\sin(17°)\hat{y}$ (note the signs). The drag force must be the exact opposite of this, so $\vec{F}_d = -\vec{F}_B = 5N\cos(17°)\hat{x} - 5N\sin(17°)\hat{y}$.

Similarly, the lift provided by the bird's wings must be exactly equal in size, but opposite in direction, to the bird's weight. We know the bird's mass is $2.7kg$ and the weight of any object near the surface of the Earth is equal to its mass times the strength of gravity, g, so $\vec{W} = -2.7kg \times 9.8\frac{m}{s^2}\hat{z} = -26.46N\,\hat{z}$. This allows us to write the lift provided by the bird's wings as $\vec{F}_L = -\vec{W} = 2.7kg \times 9.8\frac{m}{s^2}\hat{z} = 26.46N\,\hat{z}$.

2. **A rocket is launched at an angle of 33° relative to the horizontal. It has a mass of $7kg$. Consider its mass to be constant (this is usually not a good approximation for rockets, but we'll go with it this time). Its initial speed is $v = 72\frac{m}{s}$ and its engine exerts a constant force (after providing whatever was necessary to give it its initial velocity) of $\vec{F}_R = 13N\hat{x} + 5N\hat{y}$. Neglect air resistance.**
    a. **What maximum height will it reach?**
    b. **How long after launch will it strike the ground?**
    c. **How far from its launch position will it be when it strikes the ground?**
    d. **What will its velocity be when it strikes the ground?**
    e. **What will its speed be when it strikes the ground?**

One step at a time! Always break problems up into bite-sized pieces. What's happening here? The rocket is launched and has some initial velocity, it starts going up and away at that velocity. But it's got an engine which *continues* to push it after its launch, so it doesn't just fly like the baseball or the bomb we saw in previous problems. We'll need to do some work on this one. Hey, I've got a novel idea: Let's draw a picture and make a free body diagram! (Wow, I bet that came as a surprise, huh?)



Now, my amazing psychic abilities tell me that some of you took the perfectly good definition of $\vec{F}_R$ and turned it into something with an angle in it or, even worse, somehow combined the components to make some sort of scalar. Don't do that! $\vec{F}_R = 13N\hat{x} + 5N\hat{y}$ is perfectly acceptable as is—in fact, this is the best and most useful form for a vector. Please stop trying to avoid using this form and get used to it!

Here's where I have to confess to making a little mistake in the statement of the problem (this was observed by a student in an office-hour conversation—nice job!): I stated the angle the Rocket's initial velocity makes relative to the horizontal. But I don't indicate whether "the horizontal" means the positive *x* direction or the negative *x* direction (i.e., whether the *x* component of the initial velocity is positive or negative)! I just automatically visualized it as being the positive *x* direction, but the negative *x* direction would be a perfectly valid alternate interpretation. I'm going to go ahead and solve the problem assuming that the *x* component of the initial velocity is positive (i.e., in the same direction as the *x* component of the rocket's force). I recommend that you play with this problem a bit with the force in the opposite direction and see what affect that has on the answers—it's a nice variation!

Using the value of $\vec{F}_R$ as given, we see that the *net* force in the *y* direction is $F_y = 5N - mg = 5N - 7kg \times 9.8\frac{m}{s^2} = 5N - 68.6N = -63.6N$. The net force in the *x* direction is easy since there is only the force of the rocket's engine in that direction: $F_x = 13N$. We can now use these two components to deal with the problem as two, independent (mostly) problems.

Our first challenge is to find the maximum height that the rocket will reach. There are two ways of doing this that are equally valid. One is to figure out how *long* it takes to reach its maximum height and then to use $h = \frac{1}{2}at^2 + v_i t$ to find the height. The other is to use $v_f^2 - v_i^2 = 2ad$. I think this second method is slightly easier, so let's do it that way.

In order to find the height, we'll need to find the initial velocity in the vertical direction. Using our usual trig methods, we can represent the velocity in terms of components as $\vec{v}_i = 72\frac{m}{s}\cos(33°)\hat{x} + 72\frac{m}{s}\sin(33°)\hat{y}$. All we care about at the moment is the *y* component of this. This will be the "v_i" that we use to solve for the height. Since, at the maximum height, the *y* component of the velocity is zero, we can say that $v_f = 0$ at that point. (Note that we're *only* working in the vertical direction at the moment. The horizontal component of the velocity certainly will not be zero at that point.)

So we have $-\left(72\frac{m}{s}\sin(33°)\right)^2 = 2ah$. Now, here's where an error can come in: The acceleration in the *y* direction is *not g*. Since the rocket's engine provides some lift (but not enough to completely counter gravity), the vertical acceleration is somewhat less than *g*. Using Newton's second law, we can find that $F_y = -63.6N = ma_y = 7kg \times a_y$ so

$a_y = \frac{-63.6N}{7kg} = -9.09\frac{m}{s^2}$. Inserting this for the acceleration, we have $h = \frac{\left(72\frac{m}{s}\sin(33°)\right)^2}{2\times 9.09\frac{m}{s^2}} = 84.6m$.

Now, to find the time until the rocket strikes the ground, there are a couple of ways to proceed. The most straightforward is just to solve $y_f = \frac{1}{2}at^2 + v_i t + y_i$ for *t*. Set both the initial and the final height to zero. This gives $0 = \frac{1}{2}at^2 + v_i t$. I can divide through by *t* and get the answer in a step or two, but take a look at that expression for a moment. Notice that this is a quadratic equation. A quadratic equation has two "roots"—that is, two values of (in this case) *t* that will satisfy the equation. In this case, this means that there are two times at which the rocket will be on the ground—of *course* there are: It takes off from the ground and then ends up at the ground! So one root is just $t = 0$. This is trivial, of course, but it's worth noting. Anyway, divide the above equation by *t* and you get, after a tiny bit of algebra, $t = -\frac{2v_i}{a}$. (Don't worry about that minus sign. That's jus there because the initial acceleration is in the opposite direction to the initial velocity.)

There's a very important additional result buried in the above equation. Let's ask a different question: "How long will it take the rocket to reach its apex?" To solve this question, we just realize that the apex will be reached when the *y* component of the velocity is zero. Since, from the definition of acceleration, $\Delta v = at$, if the final velocity (in the *y* direction) is zero, we will have $t = -\frac{v_i}{a}$. Notice that this is exactly half of the time for the entire trip from launch to landing. Therefore, the time spent going up *must* be exactly the same as the time spent coming down! While seemingly trivial, this result is of profound importance. It seems obvious and it certainly makes intuitive sense, but it's useful to have it proven.

Anyway, finding the time for the entire trip, we get $t = -\frac{2v_i}{a} = -\frac{2\times 72\frac{m}{s}\sin(33°)}{-9.09\frac{m}{s^2}} = 8.63s$.

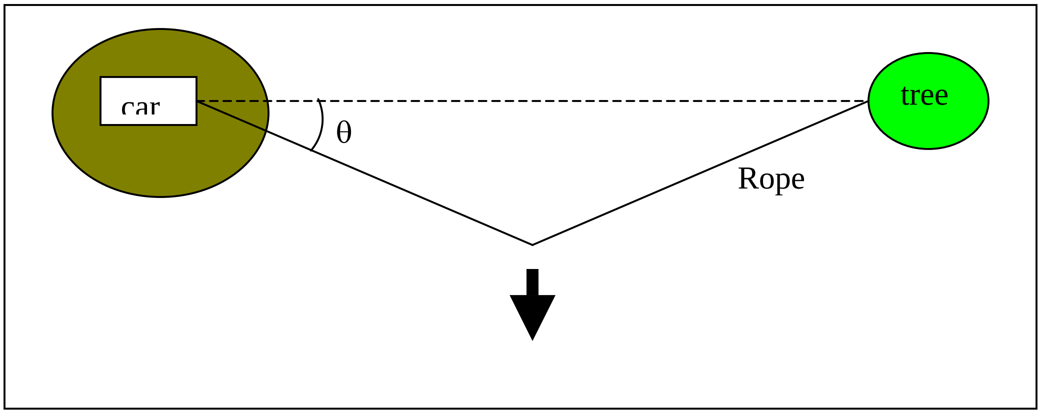
We can now use this answer to find the distance the rocket will be from its initial position when it strikes the ground. Here's a difference between this problem and others that you've done, like the baseball problem in the previous assignment: Up until now, the acceleration has only existed in one dimension while the velocity may have been present in two or even three. In this problem, we have both an acceleration and an initial velocity in both dimensions. What shall we do? No problem! Since we broke the motion up into two, independent motions, all we need to do is find the distance traveled by an object with an initial velocity of $v_i = 72\frac{m}{s}\cos(33°)$ (the x component of the velocity found above) with an acceleration of a_x (which we'll find in a moment) traveling for 8.63 seconds. Piece of cake! We just need to find that acceleration and we're set.

Since we know the force in the x direction, $13N$, and we know the mass of the rocket, $7kg$, the acceleration is simply $a_x = \frac{F_x}{m} = \frac{13N}{7kg} = 1.86\frac{m}{s^2}$. Using this, the total distance traveled is:

$$\begin{aligned} x_f &= \frac{1}{2}a_x t^2 + v_{ix}t + x_i \\ &= \frac{1}{2}\times 1.86\frac{m}{s^2}\times(8.63s)^2 + 72\frac{m}{s}\cos(33°)\times 8.63s \\ &= 69.26m + 521.1m = 590.4m \end{aligned}$$

To find the final velocity, we just use $\Delta\vec{v} = \vec{a}t$. Now, you're getting more sophisticated in the use of vectors, so we can start using some slicker techniques than we've been using. Instead of dealing with the two components of the velocity separately, let's just do them both at once. We have $\vec{v}_f = \vec{v}_i + \vec{a}t = \left(72\frac{m}{s}\cos(33°)\hat{x} + 72\frac{m}{s}\sin(33°)\hat{y}\right) + \left(1.86\frac{m}{s^2}\hat{x} - 9.09\frac{m}{s^2}\hat{y}\right)\times 8.63s$. Doing the arithmetic on this, we have $\vec{v}_f = \vec{v}_i + \vec{a}t = \left(60.4\frac{m}{s}\hat{x} + 39.2\frac{m}{s}\hat{y}\right) + \left(16.1\frac{m}{s}\hat{x} - 78.4\frac{m}{s}\hat{y}\right) = 76.5\frac{m}{s}\hat{x} - 39.2\frac{m}{s}\hat{y}$. Note that the y component is the *negative* of the initial y component. This should be no surprise: The speed of an object thrown straight up is the same when it lands as when it was originally thrown. This is not specific to an acceleration of g in the y direction. As long as the acceleration in the y direction is constant and the object comes to a stop at the apex, the y velocity of an object will be the same coming down as going up.

Finally, we want the speed when the object lands. Here we just use the Pythagorean theorem and get $v_f = \sqrt{\left(76.5\frac{m}{s}\right)^2 + \left(39.2\frac{m}{s}\right)^2} = 86\frac{m}{s}$.



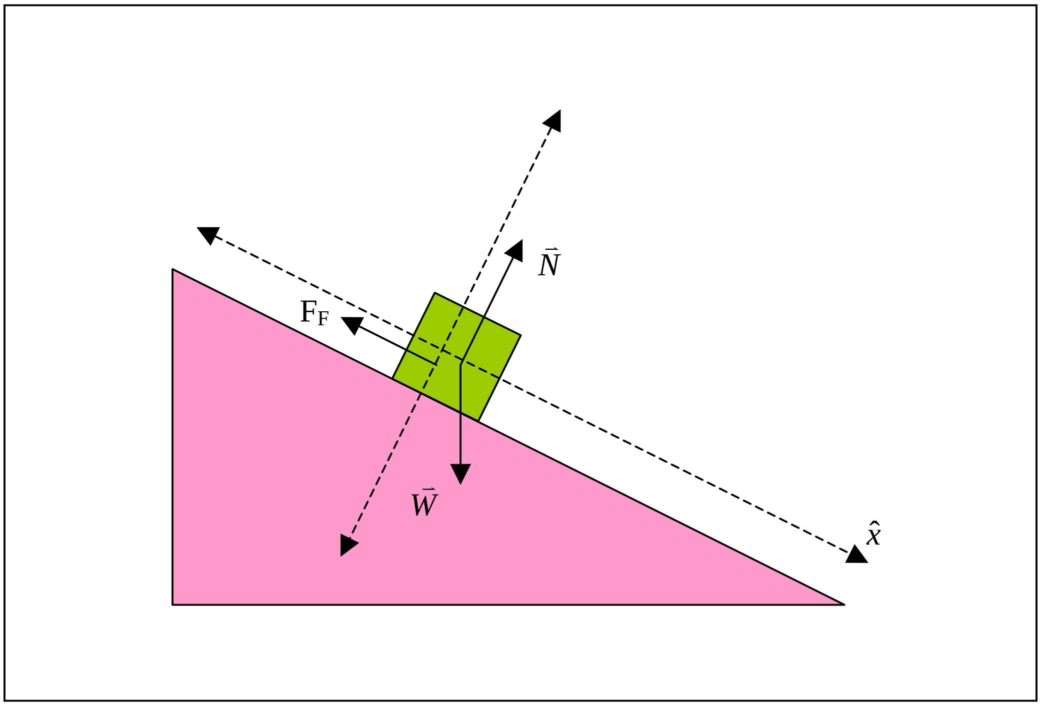
3. **A car is stuck in a mud puddle. A man wishes to pull the car out by tying a rope to a tree and to the car. He pulls at the middle of the rope. It requires a force of 10,000 Newtons to free the car from the mud. The man is capable of applying a maximum force of 1000 Newtons. What is the maximum angle θ for which the car will move?**

This is an utterly trivial problem save for the fact that I've turned it on its head for you. If I phrased it the way you're now accustomed to seeing problems, you'd have the answer in a matter of seconds. Here's the *same* problem worded in a way that you would have gotten it in a matter of seconds: "A man exerts a force of 10,000 Newtons at an angle θ relative to the *x* axis. If the *y* component of the force is 1,000 Newtons, what is θ?"

Do you see that this is the exact, same problem? It takes some doing! Let's walk through it. The rope exerts a force of 10,000 Newtons on the car. By now, you should be in the habit of decomposing every vector you see into its perpendicular components. The *x* component of this force is $F_x = 10{,}000\ N \times \cos(\theta)$. The *y* component is $F_y = 10{,}000\ N \times \sin(\theta)$. We're not quite done at this point, however. The man exerts the force on *two* ropes: One going from the car to him and the other going from the tree to him. By symmetry, he will exert the same *y* force on both ropes. (Notice that he doesn't have to exert any *x* force at all. The nature of the rope takes care of that for him.) So we have to multiply this by 2 to get the correct answer.

So $F_y = 2 \times 10{,}000\ N \times \sin(\theta) = 1{,}000\ N$. Thus $\sin(\theta) = \frac{1000}{20000} = 0.05$ which gives $\theta = 2.9°$. (Note: A lot of effort can be saved by abandoning the cumbersome system of measuring angles in degrees. A *very* powerful approximation tool exists if you measure angles in radians: The sine of an angle is approximately equal to the angle itself for small angles. So we could have written, immediately, that $\theta = 0.05$ radians without mucking about with ancient Babylonian festival dates.)

5. A block of mass *m* sits on an incline at angle θ. The coefficient of static friction, μ_s, between the block and the incline is .4. Find the "angle of repose" of the block. That is, the maximum angle at which the block will not slip.



This is right out of a lab exercise that you will do soon. The free-body diagram is shown above. Note that the direction of the frictional force is always going to be *opposite* to the direction the object wants to move. In this case, since it's trying to slide down the ramp, the friction is pointed toward the top of the ramp.

The decomposition of the forces was done in the solutions to assignment #4, so I will not reproduce it here. Simply note that the only difference is the addition of the frictional force, which has only an $\hat{x}$ component. When the ramp is exactly at the angle of repose, the frictional force will exactly balance the $\hat{x}$ component of the weight. So, we can write

$$W_x = W\sin(\theta) = F_F$$

$$W_y = W\cos(\theta) = N$$

Now, recall that $F_F \leq \mu_s N$. The "less than or equal to" symbol is crucial in this: At any angle other than the angle of repose, the frictional force will be less than its maximum value. Indeed, imagine the extreme case when θ=0. In that case the frictional force would also be zero. It is very important to remember that the equality only holds when the object is *just* about to slide. Since this is the precise definition of the situation when the ramp is at the angle of repose, we are safe to write $F_F = \mu_s N$, but, from the force decomposition for *y*, we can rewrite this as

$F_F = \mu_s N = \mu_s W\cos(\theta)$. We put this relation back into the force decomposition for *x*, which gives $W\sin(\theta) = F_F = \mu_s W\cos(\theta)$. We can divide through by $W\cos(\theta)$ to get $\mu_s = \frac{W\sin(\theta)}{W\cos(\theta)}$.

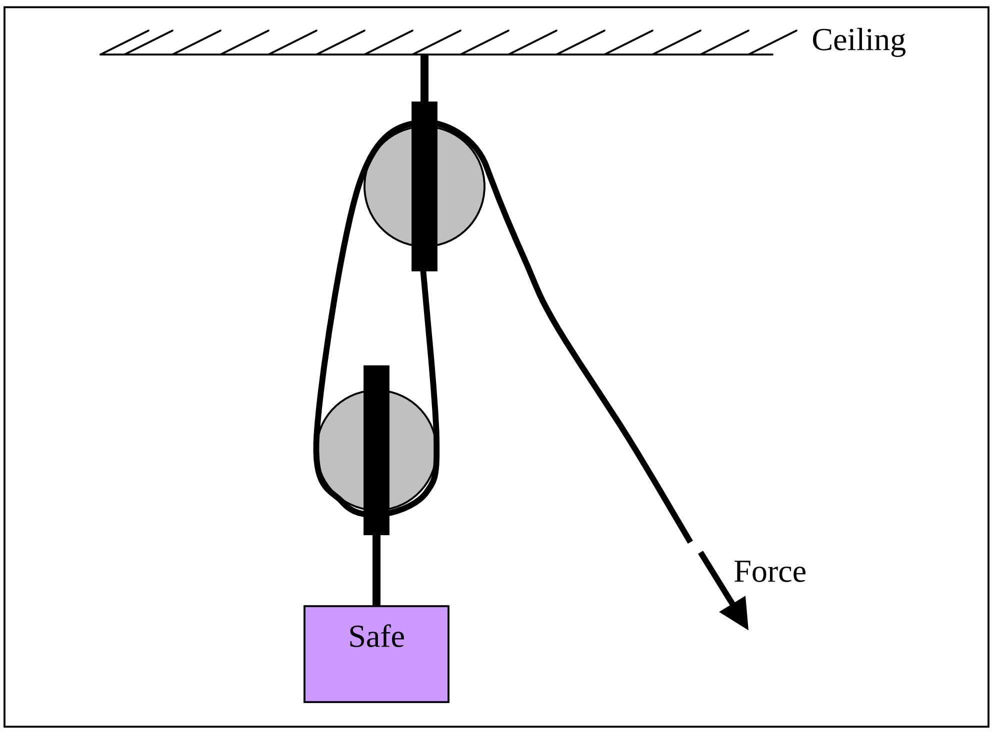
We're not quite done. Those of you who remember trigonometry recognize the relation above. For those who don't, a little algebra will help. Recall our mnemonic: SOHCAHTOA. Let's divide the sine by the cosine: $\frac{\sin}{\cos} = \frac{\frac{O}{H}}{\frac{A}{H}}$. A quick piece of algebra gives $\frac{\sin}{\cos} = \frac{\frac{O}{H}}{\frac{A}{H}} = \frac{O}{H} \times \frac{H}{A} = \frac{O}{A}$. But opposite-over-adjacent is just the definition of tangent. So we can write, for any angle, $\frac{\sin(\theta)}{\cos(\theta)} = \tan(\theta)$. Thus $\mu_s = \frac{W\sin(\theta)}{W\cos(\theta)} = \tan(\theta)$.

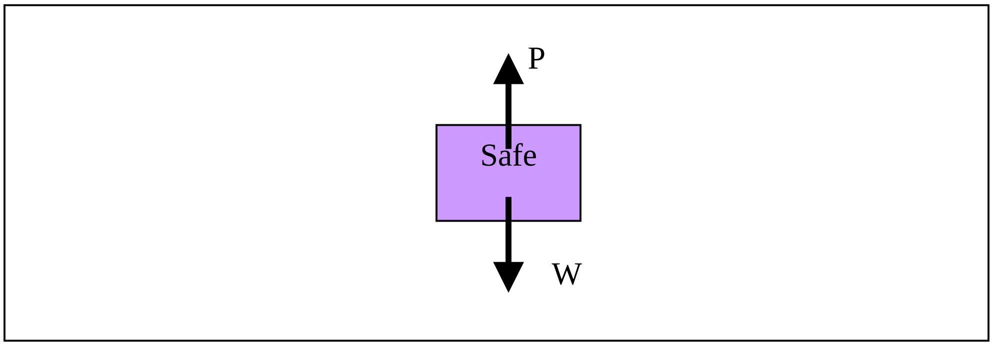
So the angle of repose will be the angle whose tangent is the coefficient of static friction. Now, for those for whom trigonometry is unfamiliar: "the angle whose tangent" is called the "arc tangent" (that's just its name—don't get obsessed with why it's called what it's called). On your calculator this will be written "$\tan^{-1}$" or "atan" or just "inverse tan". We want the arc tangent of .4. Plugging the numbers into my calculator, I get $\theta = 21.8°$, or, using radians (the preferred system for measuring angles), $\theta = 0.38\ \text{rad}$.

6. **A pair of frictionless pulleys is used in combination as shown below. The pulley combination is used to lift a safe which has a mass of 175 kg. (Hints: a) You can't push on a rope! b) The tension in a rope is the same everywhere along its length provided there is no friction in the pulley and the pulley is massless.) Note: The pulleys rotate freely within brackets. The safe is suspended from a bracket and one end of the rope is attached to the other bracket.**

   a. **Draw a free-body diagram indicating all relevant forces on the safe.**

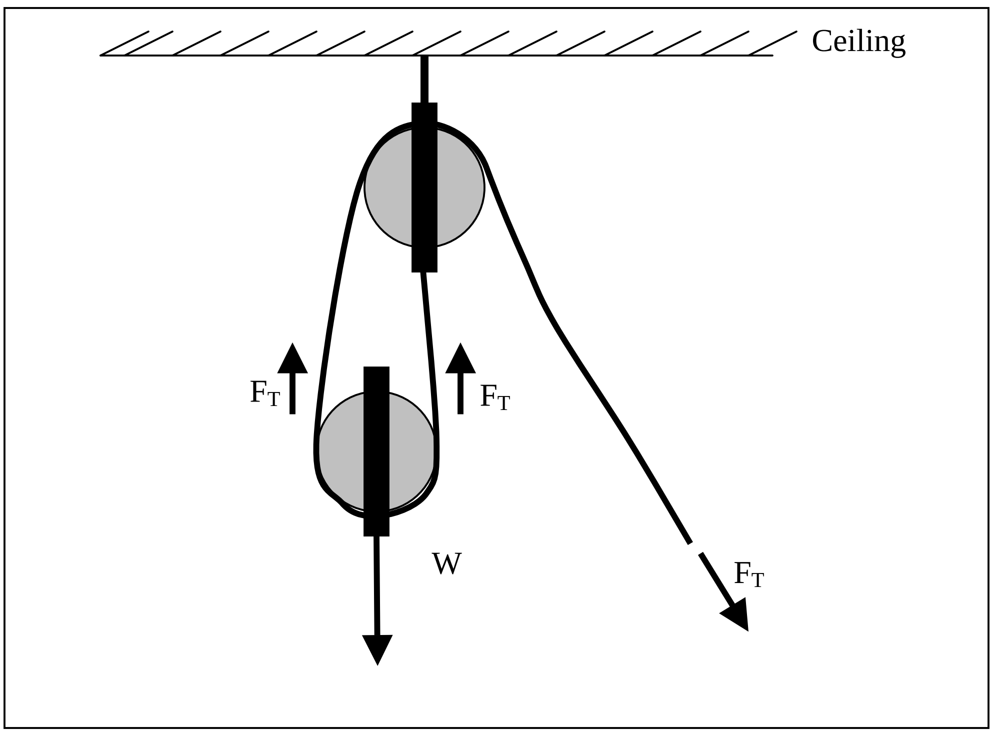
   b. **What is the force that someone needs to exert on the rope to lift the safe? Justify your answer by reference to forces indicated on your diagram.**





There are only two forces acting on the safe: The upward force exerted by the pulley (which I have labeled "P" in the free-body diagram) and the force of gravity pulling down—the weight.

Now, to figure out the force exerted on the rope, we must take this to the next step. Note that the force between the pulley and the safe is an internal force, so we don't need to include it. We can draw the following free-body diagram:



Remember my advice: You can't push on a rope! The rope passing through the lower pulley *pulls* on both sides of it. Since the tension on the rope is the same everywhere, it pulls with the same force on both sides. These two tensions are the only upward force on the pulley, so we can write $P = 2F_T$. And, from above, we can conclude that $F_T = \frac{W}{2}$. Again, since the tension is the same everywhere on a string (*if* there is no friction in the pulleys!), this is the same as the force that someone would need to exert to lift the safe.

(For a fun extra challenge: Calculate the forces experienced by the *top* pulley!)