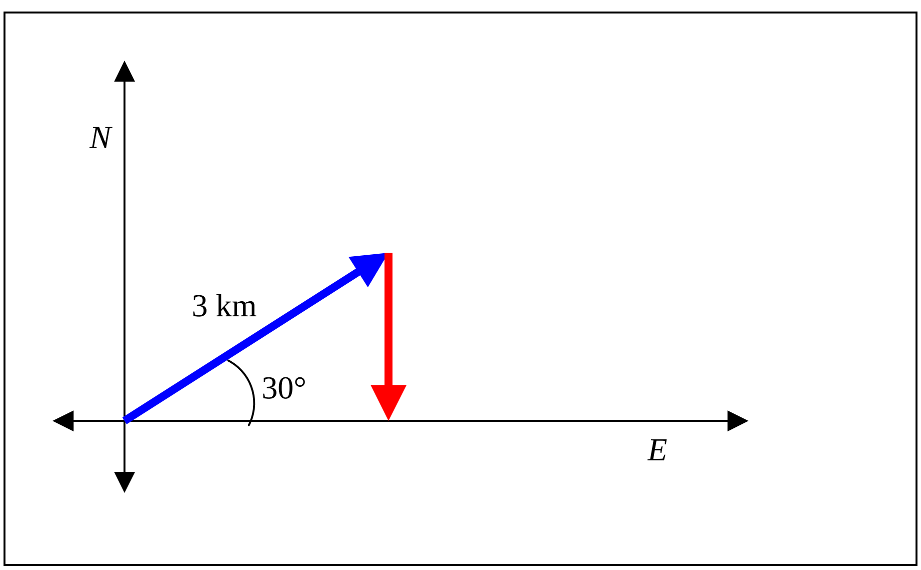
PHYSICS 206a
HOMEWORK #4
SOLUTIONS

1. **Starting with Newton's second law, determine the unit of force in the SI system. Express this in terms of the fundamental units of that system. This unit is called the "Newton" and is abbreviated by the letter "N".**

Newton's second law is just the equation $\vec{F} = m\vec{a}$. Let's invent a bit of notation to do this problem. Let's indicate the units of a quantity by putting that quantity in square brackets. For example, if we want to say "the units of velocity" we would write $[\vec{v}] = \frac{\text{meters}}{\text{second}}$. Using this notation, we want to know $[\vec{F}]$. Since units follow the rules of algebra, we can simply write $[\vec{F}] = [m][a]$. Since $[m] = \text{kilograms}$ and $[\vec{a}] = \frac{\text{meters}}{\text{second}^2}$, the unit of force is $[\vec{F}] = \text{kilogram}\frac{\text{meters}}{\text{second}^2}$. So $1\ N = 1\,\text{kilogram}\frac{\text{meter}}{\text{second}^2}$. (Some of you may be wondering about the use of the plural in units. E.g., I switched from "meters" to "meter" above. Really, there is no solid rule on this. Since "meter" is an English word, it would sound odd to leave the plural off. It makes no difference and not too much effort should be spent thinking about it.)

2. **A car travels at $26\frac{\text{meters}}{\text{second}}$ oriented 30 degrees north of east. It travels for 3 kilometers. It then turns so that it is oriented to the south. It travels at the same speed. If another car departs from the same point as the first one at the same time as the first one and travels directly east, how fast does it have to travel if the two cars are to collide?**



Clearly, the second car will travel due East only. So the collision can only occur when the first car crosses the *East* axis. Thus, the first thing we want to know is where on the *East* axis this will be—let's call that *x*. From the picture, we see that this will occur at $x = 3\text{ km} \times \cos(30°) = 2.6\text{ km}$.

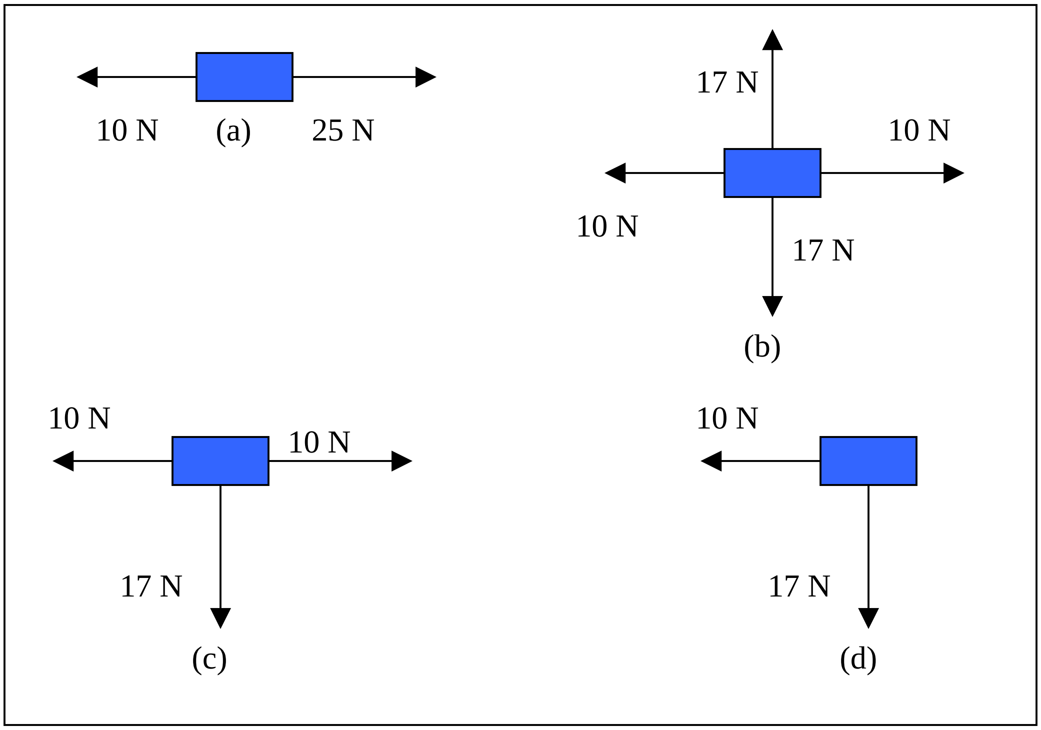
Likewise, the collision can only occur at the time the first car crosses the *East* axis. Its trip has two segments. The first one is at the 30° orientation and the second one is due South. Let's solve for the time for each segments separately. From the definition of speed $v = \frac{s}{t}$ (using *s* to stand for the distance traveled), we can write $t = \frac{s}{v}$ using a distance of 3 km and a speed of $26\,\frac{\text{meters}}{\text{second}}$, this gives $t = \frac{s}{v} = \frac{3000\text{ meters}}{26\,\frac{\text{meters}}{\text{second}}} = 115.4\text{ seconds}$. The second leg of the trip is due South. From the picture we see that the length of this is $s = 3\text{ km} \times \sin(30°) = 1500\text{ meters}$. Again using our definition of speed, we get the time for this leg to be 57.69 seconds. Adding the two times together, we get the total time of the trip to be 173.1 seconds.

So, the second car needs to travel 2.6 km in 173.1 seconds to collide with the first car. This gives a speed of $v = \frac{s}{t} = \frac{2600\text{ meters}}{173.1\text{ seconds}} = 15\,\frac{\text{meters}}{\text{second}}$.

3. **What is the net force acting on a hockey puck sliding on the ice at a constant velocity of $10\,\frac{m}{s}$ in the $\hat{x}$ direction? Explain your answer in terms of Newton's laws.**

This is so straightforward that it's almost confusing. Scientific predictions don't just work in one direction. They work both ways. This is a very important feature of Science. In other words, if a theory predicts that a certain situation will result in some particular outcome, it *also* requires that observation of the outcome allows one to presume the cause. This is an example of that principle: Newton's first law predicts that an object in motion will remain in motion in a straight line at a constant speed unless a net external force acts on it. Therefore, if we observe something to move in a straight line at a constant speed we can infer that there is no net force acting on it. Newton's second law puts this in quantitative form. (Newton's second law does more than just allow us to attach a number to the predictions of Newton's first law. It also states that the response of an object to a force does not depend on the particular nature of the force or the object. Only the size and direction of the force and the mass of the object matter. This seems like a trivial statement to our modern perspective, looking at it 319 years after Newton published it, but it's quite profound historically. Prior to Newton, the fact that an apple's fall and the orbital motion of the moon lacked any significant difference would have been considered unthinkable!) The net force acting on the puck is zero.

4. **Find the magnitude and direction of the net force on the object in each of the free-body diagrams in the figure below.**



I think most people got this one without too much effort: Just subtract forces pointed in opposite directions. Forces (indeed, all vectors) which act perpendicularly to each other act independently of each other. The results are:

a) $\vec{F} = 25\ N\hat{x} - 10\ N\hat{x} = 15\ N\hat{x}$

b) $\vec{F} = 10\ N\hat{x} - 10\ N\hat{x} + 17\ N\hat{y} - 17\ N\hat{y} = 0$ (no net force)

c) $\vec{F} = 10\ N\hat{x} - 10\ N\hat{x} - 17\ N\hat{y} = -17\ N\hat{y}$

d) $\vec{F} = -10\ N\hat{x} - 17\ N\hat{y}$ (Nothing to be done here—two perpendicular forces simply added together.)

5. **Refer again to the figure in problem #4. If the objects all have a mass of 1.7 kg, what is the acceleration experienced by them in each case?**

Here we apply Newton's second law: $\vec{F} = m\vec{a}$. Simply divide each force by m to find the acceleration.

a) $\vec{a} = \dfrac{15\ N\hat{x}}{1.7\ kg} = 8.82\ \dfrac{\text{meters}}{\text{second}^2}\,\hat{x}$

b) No net force so no acceleration.

c) $\vec{a} = \dfrac{-17\ N\hat{y}}{1.7\ kg} = -10\ \dfrac{\text{meters}}{\text{second}^2}\,\hat{y}$

d) $\vec{a} = \dfrac{-10\ N\hat{x} - 17\ N\hat{y}}{1.7\ kg} = -5.88\dfrac{\text{meters}}{\text{second}^2}\,\hat{x} - 10\dfrac{\text{meters}}{\text{second}^2}\,\hat{y}$

6. **Yet again, refer to the objects shown in problem #4. After 13 seconds, what is the velocity of each object? At that same time, what is the position vector (assuming the objects all begin at the origin) for each object?**

For the velocity, since the acceleration is constant, we can simply use the definition of acceleration $\vec{a} = \dfrac{\Delta\vec{v}}{t}$. That is, the acceleration is the change in the object's velocity divided by the time over which that change occurred. In this case, the change occurs over 13 seconds. Now, an important point which I'm sure most of you missed: Nothing was stated about the original velocity! If you assumed that the initial velocity is zero, then this assumption needs to be stated in your solution. The final velocity is the initial velocity added to the change in velocity. Note that the fact that the objects start at the origin says nothing about their velocities! Be careful! So $\vec{a} = \dfrac{\vec{v}_{final} - \vec{v}_{original}}{t}$ and a little algebra gives us $\vec{v}_{final} = \vec{a}t + \vec{v}_{original}$. Taking the time to be 13 seconds and using the accelerations found in the previous problem, this gives

a) $\vec{v}_{final} = \vec{a}t + \vec{v}_{original} = 8.82\ \dfrac{\text{meters}}{\text{second}^2} \times 13\ \text{second}\ \hat{x} + \vec{v}_{original} = 114.7\ \dfrac{\text{meters}}{\text{second}}\ \hat{x} + \vec{v}_{original}$

b) $\vec{v}_{final} = \vec{v}_{original}$

c) $\vec{v}_{final} = \vec{a}t + \vec{v}_{original} = -10\ \dfrac{\text{meters}}{\text{second}^2} \times 13\ \text{second}\ \hat{y} + \vec{v}_{original} = -130\ \dfrac{\text{meters}}{\text{second}}\ \hat{y} + \vec{v}_{original}$

d)
$$\begin{aligned}\vec{v}_{final} &= \vec{a}t + \vec{v}_{original} = -5.88\dfrac{\text{meters}}{\text{second}^2} \times 13\ \text{second}\,\hat{x} - 10\dfrac{\text{meters}}{\text{second}^2} \times 13\ \text{second}\ \hat{y} + \vec{v}_{original} \\ &= -76.44\dfrac{\text{meters}}{\text{second}}\,\hat{x} - 130\ \dfrac{\text{meters}}{\text{second}}\ \hat{y} + \vec{v}_{original}\end{aligned}$$

Next, we must find out where these objects are after 13 seconds. The total motion will consist of two parts: The motion due to the original velocity, which the object would have whether it accelerates or not, and the motion due to the "extra" velocity it "gains" due to the acceleration. I put those words into quotes since the acceleration can be in the direction opposite to that of the original velocity. Thus, the "extra" and "gain" would be negative. Since we are told that they start at the origin, we don't need to keep track of an initial position—at least one thing off our minds!

We have a formula for dealing with the distance objects travel while accelerating. It is $x = \frac{1}{2}at^2 + v_{original}\, t$. Note that I've left the original position off since we don't have one in this problem. Also, I've written this down in one dimension only. If we have an original velocity or an acceleration in two dimensions, we do what we always do with two-dimensional vectors: Break them into perpendicular components and then treat each one independently as a one-dimensional problem. Remember: Vectors which are perpendicular to each other are independent of each other!

Using this formula and the values for the acceleration found in the previous problem, we have

a)
$$\vec{D} = \tfrac{1}{2}\vec{a}t^2 + \vec{v}_{original}t = 4.41\,\frac{\text{meters}}{\text{second}^2}\times 169\text{ second}^2\ \hat{x} + \vec{v}_{original}\times 13\text{ seconds}$$
$$= 745.3\text{ meters }\hat{x} + \vec{v}_{original}\times 13\text{ seconds}$$

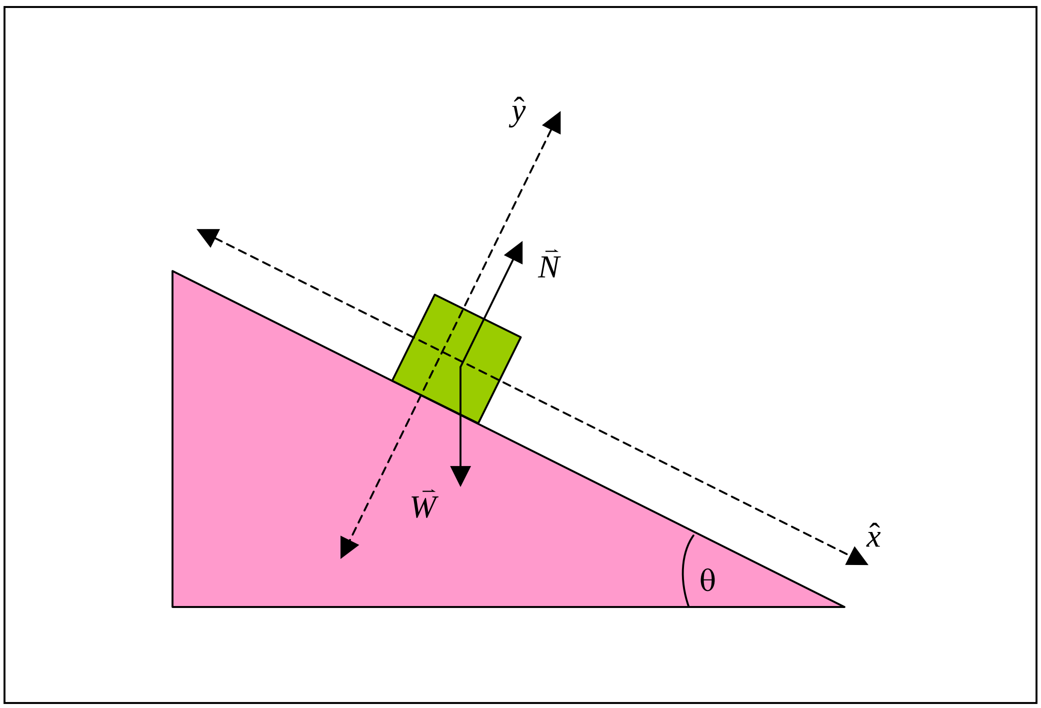
b)
$$\vec{D} = \vec{v}_{original}\times 13\text{ seconds}$$

c)
$$\vec{D} = \tfrac{1}{2}\vec{a}t^2 + \vec{v}_{original}t = -5\,\frac{\text{meters}}{\text{second}^2}\times 169\text{ second}^2\ \hat{y} + \vec{v}_{original}\times 13\text{ seconds}$$
$$= 845\text{ meters }\hat{y} + \vec{v}_{original}\times 13\text{ seconds}$$

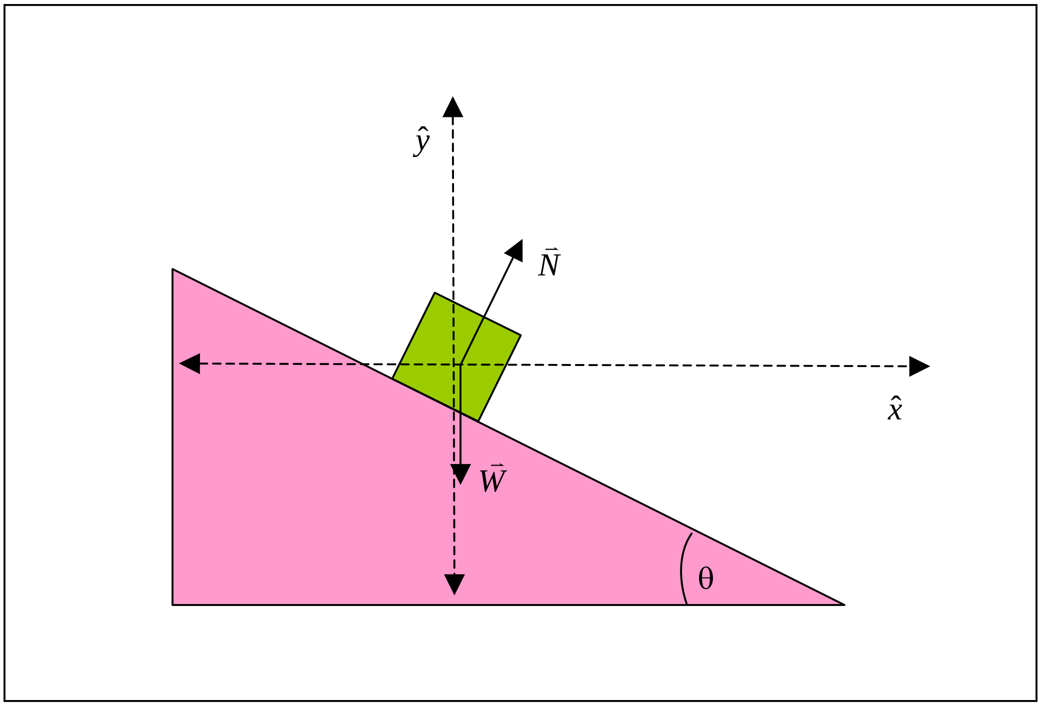
d)
$$\vec{D} = \tfrac{1}{2}\vec{a}t^2 + \vec{v}_{original}t$$
$$= -2.94\,\frac{\text{meters}}{\text{second}^2}\times 169\text{ second}^2\ \hat{x} - 5\,\frac{\text{meters}}{\text{second}^2}\times 169\text{ second}^2\ \hat{y} + \vec{v}_{original}\times 13\text{ seconds}$$
$$= -497\, meters\ \hat{x} - 845\text{ meters }\hat{y} + \vec{v}_{original}\times 13\text{ seconds}$$

7. **In your lab exercise (#3 on Jan. 29-30), you place a mass on an inclined airtrack. In class, I will derive the acceleration of the mass using a coordinate system with one axis parallel to the track. Derive an expression for the acceleration of the mass (the cart) as a function of the angle of the incline using the "usual" coordinate system, i.e., one in which the *y* axis is in the direction of gravity. Draw a free-body diagram of the cart and determine the forces (both size and direction) in this coordinate system. You may consider the mass of the cart to be simply "*m*".**

A colleague of mine saw this problem and said "ah, you're using the same technique I used to train my dog not to go near the hot stove: You're letting them get burned so the pain educates them!" Indeed, that *is* my primary goal in this. The choice of an inappropriate coordinate system can render a trivial problem all but unworkable.



Before we discuss how to solve this problem using the "wrong" coordinate system, let's see how we can know which system to use. I've drawn the system above using the "right" coordinate system. Notice that, by the nature of the problem, the mass is constrained to remain on the surface of the incline. Any motion of the mass, therefore will be in the $\hat{x}$ direction, if we choose it to be parallel to the surface. Our problem will only require one force and one acceleration. Compare this with the same problem using the "wrong" coordinate system:

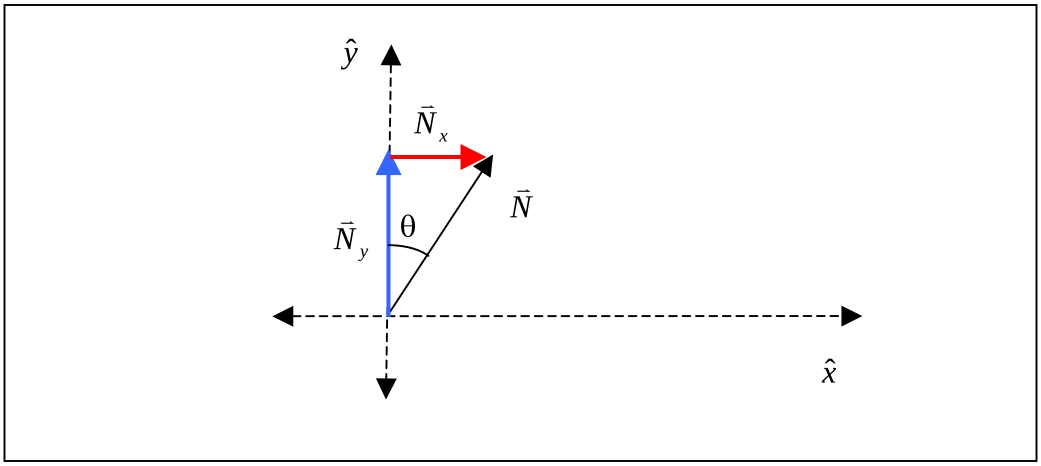


Now, a point on the surface of the incline will necessarily be described by both an x and a y coordinate. All motion along the surface will be 2-d. We've got a big mess on our hands!

Let's dig in and see where this takes us. As discussed above, we'll have to decompose the acceleration into two components since the motion is constrained to the surface of the incline. Before we do this, let's decompose the forces into their components. The weight is all in the $\hat{y}$ direction, so that's easy. The normal force, on the other hand, makes an angle θ relative to the y axis. (Prove this to your satisfaction. Come see me if you're not certain about this—perhaps I'll write up the proof for you and post it later.)

Let's discuss the normal force for a moment. The Normal force is a weird thing. It's a catch-all for what is actually a whole set of forces. But, in a macroscopic problem like this one, we can lump them all together. Basically, the Normal force is the force which acts perpendicularly to a surface (hence the name) in reaction to the force exerted on that surface by some other object. The important features of the Normal force are 1) that it acts exactly perpendicular to a surface and 2) that it takes whatever size it needs to to counter some other force. For example, if you lean on a wall, you exert a force on the wall. More importantly, the wall exerts a force on you. That force is exactly the same size as the force you exerted on the wall in the first place, no more and no less. (This is a result of Newton's Third Law which we haven't discussed yet but will get to soon.)

Decomposing the normal force, we have the situation picture below.



Now, notice a possible source of error here: We usually define our angles relative to the *x* axis. In this case, we happen to know an angle relative to the *y* axis, so we'll use this instead. This means that the *x* component will depend on the sine of the angle, rather than the cosine and similarly for the *y* component. We certainly *could* use our familiar decomposition method—we'd just have to use $90° - \theta$ as our angle. But this is an extra bit of work. I'll stick with the angle we've got. We have

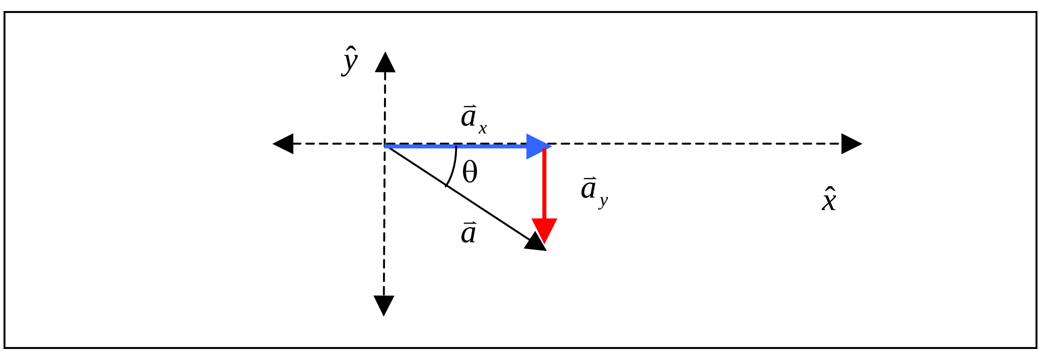
$$N_x = N\sin(\theta)$$
$$N_y = N\cos(\theta)$$

Adding forces together, we have

$$F_y = N_y - W = N\cos(\theta) - W$$
$$F_x = N\sin(\theta)$$

We now have two equations but three things that we don't know (F_x, F_y and *N*). The rule is that we need to have as many equations as unknowns if we are to be able to solve the system. We'll need something else. Fortunately, we do have something else. We have a relation for the acceleration.

The acceleration, as stated above, is constrained to the surface of the incline. So we must have the situation depicted below:



Notice the for the acceleration, we do have the usual decomposition

$$a_x = a\cos(\theta)$$
$$a_y = -a\sin(\theta)$$

Here we've had to be very careful about the sign on the *y* component: We're taking θ to be a positive angle, since that's what it was defined as initially. But for the decomposition of the acceleration, the angle appears in the negative sense. Don't apply "formulas" blindly!

Now we can use Newton's second law to relate the forces to the accelerations:

$$F_y = N_y - W = N\cos(\theta) - W = -ma\sin(\theta)$$
$$F_x = N\sin(\theta) = ma\cos(\theta)$$

Notice that we now have two equations and only *two* unknowns: *a* and *N*. We don't care about *N*, so let's rearrange the second of these two equations to solve for *N* (I know, that sounds like the exact opposite of what you want, but follow along). We can then substitute the result for *N* in the first equation. Here we go:

$$N = ma\frac{\cos(\theta)}{\sin(\theta)}$$

Which gives

$$ma\frac{\cos(\theta)}{\sin(\theta)}\cos(\theta) - W = -ma\sin(\theta)$$

Hey! Look what we've got: One equation with only one unknown—*a*. This is what we want. So let's rearrange the equation to solve for *a*. I'll do this step-by-step, but without narrative since you should be able to follow the algebra:

$$ma\frac{\cos(\theta)}{\sin(\theta)}\cos(\theta) + ma\sin(\theta) = W$$

$$W = mg = ma\left(\frac{\cos^2(\theta) + \sin^2(\theta)}{\sin(\theta)}\right)$$

$$a = g\left(\frac{\sin(\theta)}{\cos^2(\theta) + \sin^2(\theta)}\right)$$

For the final step, we'll need a relation taught early on in a Trigonometry course but which I haven't taught you yet. Let's use our definitions of the trig functions: SOHCAHTOA and add $\sin^2(\theta)$ and $\cos^2(\theta)$. We have

$$\cos^2(\theta) + \sin^2(\theta) = \frac{A^2}{H^2} + \frac{O^2}{H^2} = \frac{A^2 + O^2}{H^2}$$

But, the theorem of Pythagoras says that $H^2 = O^2 + A^2$, so $\cos^2(\theta) + \sin^2(\theta) = \dfrac{A^2 + O^2}{H^2} = \dfrac{H^2}{H^2} = 1$ for all angles! This is a very powerful relation which I recommend you memorize.

Using this, we can write

$$a = g\left(\frac{\sin(\theta)}{\cos^2(\theta) + \sin^2(\theta)}\right) = g\sin(\theta)$$

This is exactly the same as we found using the "right" coordinate system, as it must be, but *way* more work!

8. **An airplane is flying at a height of 50,000 meters at a constant velocity of $260\,\dfrac{\text{meters}}{\text{second}}$ in the $\hat{x}$ direction. A bomb is dropped out of it. The bombardier wishes to strike a target which is at $x = 100{,}000$ meters. At what x coordinate should the airplane be for the bomb to hit this target? Assume no air-resistance for the bomb. Explain your answer.**

To do this problem, we recall a rule that I've stated many times this semester: Vectors which are perpendicular to each other are independent of each other! We break vectors apart into components which are perpendicular to each other specifically so that we can take advantage of this rule. In this case, the velocity of the bomb has an $\hat{x}$ and a $\hat{y}$ component. We deal with these separately and each will give us specific information about the problem.

Before the bomb is dropped, it has whatever velocity the airplane has relative to the ground. Looked at a different way, in a coordinate system fixed to the airplane (which is an inertial frame of reference since it is not accelerating), the bomb has a velocity of zero before it is dropped. According to Newton's first law, it will retain this velocity unless a net, external force acts on it. Thus, unless a net, external force acts on the bomb, it will always have a velocity of zero in the coordinate system of the airplane.

When the bomb is dropped, it has only one force acting on it: That of gravity. It, therefore, accelerates. Gravity acts only downward—in the $-\hat{y}$ direction. So, from the perspective of the ground, the airplane will have a velocity with an $\hat{x}$ component which doesn't change and a $\hat{y}$ component which increases downward by $9.8\,\dfrac{\text{meters}}{\text{second}^2}$. From the perspective of the airplane, the bomb *still* has no velocity in the $\hat{x}$ direction—there's no acceleration in that direction. The acceleration will be the same as that seen from the ground: $9.8\,\dfrac{\text{meters}}{\text{second}^2}$ in the downward direction. (Note that this is a general rule: Changing from one coordinate system to another can change velocities, but it will never change acceleration as long as the coordinate systems are not accelerating. Non-accelerating coordinate systems are called "inertial." As we've discussed many times this semester, we are totally free to pick any inertial coordinate system we like. There is no experiment we can perform which can conclude that one is "right" and another one is "wrong." We can never say that one is moving and the other is standing still. On the other hand, if we have one system which is accelerating and another one which is not accelerating, we *can* determine which one is accelerating and which one is not. Further, if *both* are accelerating, we can determine each ones acceleration in an absolute sense. We will explore this in a future problem set.)

We can use the acceleration in the $\hat{y}$ direction and the fact that we know the initial height of the airplane to calculate the time it will take for the bomb to hit the ground. Since it has no initial velocity in the $\hat{y}$ direction, we can write $y = \frac{1}{2}at^2 = h$ where h is the height of the airplane at the instant the bomb is released. Using $-9.8\,\frac{\text{meters}}{\text{second}^2}$ for a and $-50{,}000\text{ meters}$ for h (the acceleration is negative because it's pointing downward; we use a negative number for the height since we're interested in the time to go *down* that distance but "height" is usually measured upward) we have

$$t = \sqrt{\frac{2h}{a}} = \sqrt{\frac{2 \times 50000\text{ meters}}{9.8\,\frac{\text{meters}}{\text{second}^2}}} = 101\text{ seconds}.$$

Note how large this is. I used a realistic number for the height of the airplane—maybe not for a bomber, but for a passenger jet. This is more than 1.5 minutes of falling! (Indeed, in a real situation, air resistance would make it fall even longer.) The airplane will travel a really long distance in that time. Since the $\hat{x}$ component of the bomb's velocity is unaffected by the acceleration, the bomb will travel the same distance forward. Assuming the airplane has a velocity of $\vec{V}_A = 260\,\frac{\text{meters}}{\text{second}}\hat{x}$, the distance the bomb will travel in the $\hat{x}$ direction in a time t is just $d_x = V_A t$. So, the airplane must release the bomb $d_x = V_A t = 260\,\frac{\text{meters}}{\text{second}} \times 101\text{ seconds} = 2.626 \times 10^4\text{ meters}$ before the target (this is about 16 miles—for you folks who are still uncomfortable with kilometers). Since the target is at $x = 100{,}000\text{ meters}$, this means that the airplane's x coordinate must be $x = 100{,}000\text{ meters} - 2.626 \times 10^4\text{ meters} = 7.374 \times 10^4\text{ meters}$ in order to strike the target.

9. What is the velocity with which the bomb in problem #8 will strike the target?

We know the $\hat{x}$ component of the bomb's velocity already—it's the same as that of the airplane. We can easily find the $\hat{y}$ component since we know the acceleration in the $\hat{y}$ direction and we know the time it takes to fall. We have $V_y = at = -9.8\,\frac{\text{meters}}{\text{second}^2} \times 101\text{ seconds} = -989.8\,\frac{\text{meters}}{\text{second}}$. So the total velocity on impact will be $\vec{V}_{bomb} = 260\,\frac{\text{meters}}{\text{second}}\hat{x} - 989.8\,\frac{\text{meters}}{\text{second}}\hat{y}$.

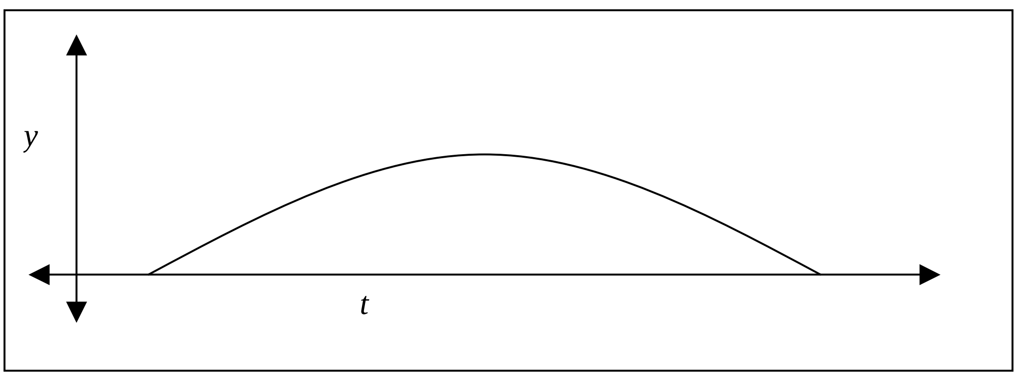
I'll reiterate something I've been beating on you about all semester: This is a vector. You can go no further. It is certainly possible to restate this in terms of an overall speed and an angle, but there is no reason to do so—this form of answer is perfectly acceptable unless you are specifically asked for something else. Whatever you do, do *not* go and turn this into a scalar! That would turn a perfectly good, correct answer into an incorrect one.

10. A baseball is thrown with an initial velocity of $40\,\frac{\text{meters}}{\text{second}}$ at an angle of 53° relative to horizontal.

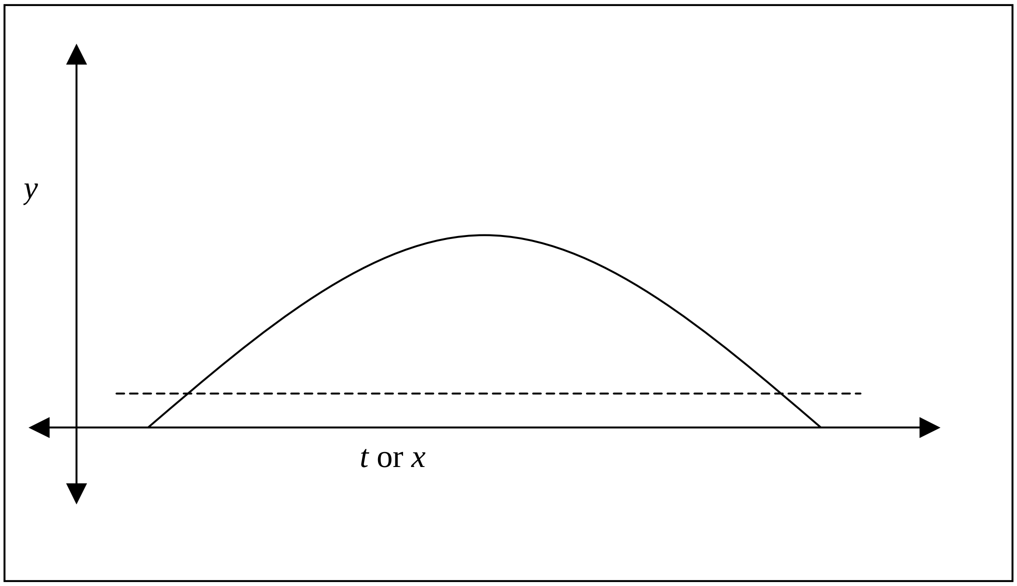
- **a. How long after it is thrown will it reach a height of 10 meters while traveling upward?**
- **b. How long after it is thrown will it reach its maximum height?**
- **c. What is its maximum height?**
- **d. How long after it is thrown will it reach a height of 10 meters while traveling downward?**
- **e. How long after it is thrown will it hit the ground?**
- **f. How far away from the thrower will it be when it hits the ground?**
- **g. What will its velocity be when it hits the ground?**

We now have a combination of several problems that we've done previously. Here we have a velocity given to us in two dimensions. We know how to deal with this, however: Break the vector up into its perpendicular components! The components can then be dealt with independently. Let's do the vertical component first.

a. Since the problem doesn't state anything to the contrary, we should assume the ball starts at a height of zero. Before solving this explicitly, let's think about its path. It will accelerate in the -*y* direction. Starting with an initial upward velocity, it will slow down, stop, and then come back down. A plot of its height as a function of time will look something like:



Interestingly, since the *x* position is going to vary linearly with time (i.e., we're going to have $x = v_x t$) a plot of *y* vs. *t* will have the identical appearance, except for a "stretching" of the *x* axis, to a plot of *y* vs. *x*, which is just a picture of the overall trajectory. (As a challenge to yourself, recast the solution for $y(t)$ into a $y(x)$ form. I considered putting this on as a homework problem and may decide to do so in the future.)



Note the figure above. We can see from the dotted line that, for a given height, there will be *two* times (or x positions) at which the ball will have the same height. This is obvious, but it's worth saying anyway. We'll see why in a minute. Let's figure out the times at which it will have a particular height. Assuming that it starts at $y_0 = 0$, we have $y = \frac{1}{2}at^2 + v_{0y}t$. Rearranging this just a bit, we have $\frac{1}{2}at^2 + v_{0y}t - y = 0$. Well, this is a quadratic equation with t serving as the unknown. We have a well-known solution for this. (If you don't remember what it is or don't feel comfortable working with it, you *must* make the effort needed to refamiliarize yourself with it. This is a minimal skill which is prerequisite for this course.) Using the "quadratic formula" we can find the values of t that make this equation true. This gives us $t = \frac{-v_{0y} \pm \sqrt{v_{oy}^2 + 2ay}}{a}$. Now, be careful: The generic form of the quadratic formula has a quantity in it called "*a*". The specific equation that we have *also* has a quantity in it called "*a*" and in *almost* the same role. Do not confuse them! They differ by a factor of ½ which can really screw things up.

Now it's just a matter of plugging things in. For v_{0y} we use SOHCAHTOA to find

$v_{0y} = v_0 \sin(\theta) = 40 \frac{\text{meters}}{\text{second}} \times \sin(53°) = 31.9 \frac{\text{meters}}{\text{second}}$. For the acceleration, we use g, being careful to use a negative sign for it since it acts in the downward direction always. Thus we have

$$t = \frac{-v_{0y} \pm \sqrt{v_{oy}^2 + 2ay}}{a}$$

$$= \frac{-31.9\ \frac{\text{meters}}{\text{second}} \pm \sqrt{\left(31.9\ \frac{\text{meters}}{\text{second}}\right)^2 - 2\times 9.8\ \frac{\text{meters}}{\text{second}^2}\times 10\ \text{meters}}}{-9.8\ \frac{\text{meters}}{\text{second}^2}}$$

Note that we used $y = 10\ \text{meters}$ since we're trying to find the time at which the ball will be at a height of 10 meters. Grind through all the numbers on this and you'll get $3.26\ \text{seconds} \pm 2.92\ \text{seconds}$. This gives us not one but *two* answers. How do we pick which one to use? In fact, we'll just pick both: Notice that part (d) of this problem asks for the time at which the ball will pass through a height of 10 meters while traveling downward. Well, our equation doesn't specify upward or downward, it just gives us the times at which the ball has a height of 10 meters. Since the ball has that height two times on this trip, the first while headed up and the other while headed back down, the equation gave us two times. (Think about that for a moment: What a remarkable tool this relation is!) The earlier (smaller) value will be the one at which it's going up and the later (larger) value will be the one at which it's coming down. These are 0.34 seconds and 6.18 seconds, respectively.

As a final step, just look at the numbers and see if they make sense. The ball starts out going up at $40\ \frac{\text{meters}}{\text{second}}$. If it didn't accelerate (remember: in the early stages it's "decelerating," but we don't use that word), it would take it ¼ second to travel 10 meters. We have a result that's somewhat larger than ¼ second, indicating that it's slowing down, but not a heck of a lot. After all, it will only slow down by 25% in the first second and this is less than one second. So our result is reasonable. It's worthwhile to get in the habit of doing this kind of test on your results.

b. It will reach its maximum height when its upward speed is zero. Using $\Delta v = at$, with gravity as the acceleration and $v_{0y} = \Delta v$, we get $t = 3.25\ \text{seconds}$.

c. To find its maximum height we just use the procedure we've used several times so far. The only difference is that we must use v_{0y} instead of the full speed. Using this, we have $v_{0y}^2 = 2gy$ which gives a total height of 51.9 meters. An alternative method would be to use $y = \frac{1}{2}at^2 + v_{0y}t$ with 3.25 seconds substituted for *t*.

d. We did this as part of (b). The ball will pass through a height of 10 meters 6.18 seconds after being thrown.

e. To find how long it will be until the ball hits the ground, we could just use $y = \frac{1}{2}at^2 + v_{0y}t$ again—this time picking $y = 0$. This would allow us to cancel a factor of *t* and we would have (after subtracting and dividing appropriately) $t = \frac{2v_{0y}}{a}$. However, we have a powerful tool at our disposal in this problem: Symmetry. The ball's path coming down will be a precise mirror image of its path on the way up. It will take just as long to come down as it did to reach the apex of its trajectory. Thus we can just *double* the result we found in part b. and state that it will hit the ground 6.5 seconds after being thrown.

f. To find the distance from the thrower the ball will have traveled before hitting the ground, we use the same strategy as we did in the airplane/bomb problem. We've already done most of the work: We know how long it took to hit the ground after being thrown. All we need to do is find the *x* component of its velocity (which will not change in this problem). This is $v_{0x} = 40\,\frac{\text{meters}}{\text{second}} \times \cos(53°) = 24.1\frac{\text{meters}}{\text{second}}$. Now, we just multiply this by the time found in e. This gives $x = v_{0x}t = 24.1\frac{\text{meters}}{\text{second}} \times 6.5\,\text{seconds} = 156.5\,\text{meters}$.

g. Finally, to find the ball's final velocity, we have several options. Soon, we will learn about conservation of energy, which will give us another option. But for now, one option is just to invoke symmetry again. We know that the ball's velocity will have the same magnitude as when it was thrown. It's direction will be a mirror image. So the final velocity will be ($127° = 180° - 53°$). Another way to find the solution is to realize, again using symmetry, that the *y* component of the velocity will be the *negative* of what it was when the ball was thrown (i.e., the ball is coming down just as fast as it went up) but the *x* component of the velocity is unchanged. This gives $\vec{v}_f = 24.1\frac{\text{meters}}{\text{second}}\hat{x} - 31.9\frac{\text{meters}}{\text{second}}\hat{y}$. A third way is to do it systematically, although this does repeat steps. In general, the velocity of the ball at any instant between being thrown and hitting the ground will be $\vec{v} = \vec{v}_0 + \vec{a}t$. Substituting the appropriate values for $\vec{v}_0$ and $\vec{a}$, we can get the velocity at any instant while it is traveling. This is, by far, the cleanest and most powerful of the methods since it doesn't rely on any special knowledge about any special point, but it may be a bit cumbersome.

The one thing which is wrong, WRONG, **WRONG** is to leave out the vector nature of the solution! The question asked for a velocity, which is a vector. Providing a scalar (i.e., the speed) answer to a question asking for a vector is simply wrong and will earn a grade of zero (as many of you will learn on the first exam).