PHYSICS 206a
HOMEWORK #2
SOLUTIONS

1. **For each of the dimensions below, express at least three different units which might be used to measure them:**
   a. **Length**
   b. **Time**
   c. **Mass**

Again, recall that "dimension" refers to the type of quantity being measured while "unit" refers to the specific device that one would use to do the measurement. There are four fundamental dimensions which will be used in this course, three of these are listed in this problem. The dimensions of all quantities which we will discuss this semester are combinations of these three fundamental dimensions.

Units have been constructed for the fundamental dimensions using many different systems around the world. Although in the United States the system that remains the most popular is the "English" system, also called the "Imperial" system, the system adopted overwhelmingly by the rest of the world is the "metric" system. Because all the units within the metric system are consistent with each other, this system has been adopted by the scientific community almost exclusively. The organization which has taken it upon itself to define this system is headquartered in Paris, France. It is the *Bureau International des Poids et Mesures*—the International Bureau of Weights and Measures. Within the metric system is a subset of units. These are, collectively, known as the *Système International d'Unités*—the International System of Units, or SI for short. In this class, we will stick to metric units almost exclusively and SI units mostly. (A link to a page containing the history of the SI units can be found on the class web site.)

The SI unit for Length is the meter. Other metric length units are: The centimeter (1/100 of a meter) and the millimeter (1/1000 of a meter). English units of length include inches, feet, and barleycorns. Other units of length include: Cubits, rods, chains, parsecs, lightyears, yards, miles, calibers, furlongs, fathoms, leagues, and hands. This list is not exhaustive!

The SI unit for time is the second. The second is also the unit of time used in the English system. Other units of time include: Minutes, hours, days, weeks, fortnights, years, centuries, generations, millennia, and shakes (as in "of a lamb's tail"—could I make something like that up? a "shake" is 1/10 of a nanosecond).

The SI unit for mass is the kilogram. As you will learn in class, "mass" is **NOT** a synonym for "weight." The two concepts are related but not equivalent. On this assignment, you will not be penalized for confusing the two, but in all future assignments (and exams) you will be expected to distinguish between them. Other units of mass (not weight) are: Grams, slugs (the English unit for mass), and AMUs (atomic mass units).

One very convenient feature of the metric system is a consistent nomenclature for fractional or multiplicative units. That is, there is a system for breaking up a unit into smaller units or multiplying to get a larger unit. In the English system, a yard is broken into three feet while a mile is broken into 5280 feet (this originated with the distance a Roman soldier walked in 1000 paces). A foot is broken into 12 inches. Inches are broken into fractions based on powers of 2, which is at least consistent at that level. In the metric system, units are broken into fractions based on powers of ten—usually, but not always, by factors of 1000. Starting with the basic unit, these would be named based on how many factors of 1000 the unit is multiplied or divided by to get the next unit—either larger or smaller. The naming is done with a prefix. The prefixes and the powers of ten associated with them are:

Prefix	Symbol	Multiply by
yotta	Y	10^{24}
zetta	Z	10^{21}
exa	E	10^{18}
peta	P	10^{15}
tera	T	10^{12}
giga	G	10^{9}
mega	M	10^{6}
kilo	k	10^{3}
No prefix		$10^{0}=1$
deci	d	10^{-1}
centi	c	10^{-2}
milli	m	10^{-3}
micro	μ	10^{-6}
nano	n	10^{-9}
pico	p	10^{-12}
femto	f	10^{-15}
atto	a	10^{-18}
zepto	z	10^{-21}
yocto	y	10^{-24}

So, a millimeter is 10^{-3} of a meter. A kilogram is 10^{3} grams, and so on. (I left a few officially-recognized prefixes off this list since they are almost never used.)

You are *required* to memorize the prefixes and associated factors in the above table for every value from "tera" to "pico"! Failure to use the correct factor for a particular prefix on an exam question will result in little or no credit for that question.

I have noticed a deeply disturbing disregard for consistency in the use of these prefixes by students in prior iterations of this course. This is terrifying in a class consisting of so many students intending to become practicing professionals in some health-related profession! The notion that I might, someday, have pharmaceuticals issued to me by someone who confuses milligrams with micrograms fills me with dread. By seeing to it that such a person fails this class, I will save lives and do the world a service.

2. **The diameter of a human hair is approximately 50 μm. Calculate what this is in inches making sure to show how the units convert.**

There are 2.54 centimeters in every inch. We write this as $2.54\,\frac{cm}{inch}$. Note that the English word "per" translates into the mathematical operation of division. So we read this as "2.54 centimeters per inch." This is a useful and powerful thing to remember! We want to know how many inches there are per centimeter, so we can directly take the reciprocal of this: $\frac{1}{2.54}\frac{inch}{cm} = .3937\,\frac{inch}{cm}$. Now we can refer to the table given in the solution to Problem #1 to find out how many centimeters there are in a micrometer (also called a "micron"). Since a cm is 10^{-2} meters and a μm is 10^{-6} meters, there are 10^{4} μm per cm. Written explicitly (this is a seriously gory level of detail that I wouldn't normally expect, but it's nice to do it as an exercise):

$$10^{-2}\,\frac{meter}{centimeter}\times 10^{6}\,\frac{micrometer}{meter} = 10^{(6-2)}\,\frac{micrometer}{centimeter} = 10^{4}\,\frac{micrometer}{centimeter}\text{, or,}$$

equivalently $10^{-4}\,\frac{centimeter}{micrometer}$. Note in the above operations how the units were simply treated as algebraic entities. I operated on them the same way as I operated on the other algebraic symbols in the equations.

So let's combine the two results we got:
$.3937\,\frac{inch}{centimeter}\times 10^{-4}\,\frac{centimeter}{micrometer} = .3937\times 10^{-4}\,\frac{inch}{micrometer}$. All that's left to do is multiply by 50 micrometers. So, we can say that the diameter of a human hair is

$$50\ micrometers\times .3937\times 10^{-4}\,\frac{inch}{micrometer} = 50\times .3937\times 10^{-4}\ inch = 1.97\times 10^{-3}\ inch.$$

Again, I wouldn't expect to see quite this much detail in a future assignment. However, you should make sure you understood every step in the above process. Even if you don't write them out, you will make all these steps (at least!) when you do problems in the future. If you don't understand every single step now, you'll have trouble in the future.

3. **Consider the hair described above. What is the area of a cross-section of one shaft of such a hair? Express your answer in square meters (m^2), square centimeters (cm^2), and square inches (in^2).**

If we assume that a shaft of hair has a circular cross-section, we can use the formula for the area of a circle: $A = \pi r^2$. Recall that the radius is one half of the diameter. Now, there are two ways to attack this problem. Both are equally valid, but one might lead to an error while the other is rather safer. Let's do the safe way first.

We were told that the diameter of the hair is 50 μm. By referring to the table in the solution to problem #1, we can write this as $D = 50\times 10^{-6}\text{ meter}$. Dividing by 2 gives $r = 25\times 10^{-6}\text{ meter}$. So $A = \pi\times(25\times 10^{-6}\text{ meter})^2$. Here's a source of error: Remember to square *everything* that is part of the radius! This includes the power of ten and the unit.

Recall the rule for raising an exponent to a power: When I raise an exponent to a power, I *multiply* the exponent by the power. So $(25\times 10^{-6}\ \text{meter})^2 = 25^2 \times 10^{-6\times 2}\ \text{meter}^2 = 625\times 10^{-12}\ \text{meter}^2$. Finally, multiply by π to give the answer (with a little bit of cleaning-up—it's not necessary to substitute an approximate value for π, but this does lead to a neater result):

$$\begin{aligned} A &= \pi \times 625\times 10^{-12}\ \text{meter}^2 = 3.14\times 625\times 10^{-12}\ \text{meter}^2 \\ &= 1963\times 10^{-12}\ \text{meter}^2 = 1.963\times 10^{-9}\ \text{meter}^2 \end{aligned}$$

Now, let's do it the dangerous way. Here, we don't convert microns to meters until the end. $A = \pi \times 625\ \mu\text{m}^2 = 3.14\times 625\ \mu\text{m}^2 = 1963\ \mu m^2 = 1.963\times 10^3\ \mu\text{m}^2$

So far, so good. I sure wish I had a dollar for every time I saw someone drop the ball at this point. Since the problem specified that the answer had to be in meters, we need to convert the μm to meter. When doing this, you *must* remember to square the factor! If you remember that $1\ \mu m^2 = \dfrac{\left(10^{-6}\ \text{meter}\right)^2}{\mu m^2} = \dfrac{10^{-12}\ \text{meter}^2}{\mu m^2}$ everything will be fine.

$$\begin{aligned} A &= 1.963\times 10^3\ \mu\text{m}^2 = 1.963\times 10^3\ \mu\text{m}^2 \times \frac{10^{-12}\ \text{meter}^2}{\mu m^2} \\ &= 1.963\times 10^{3-12}\ \text{meter}^2 = 1.963\times 10^{-9}\ \text{meter}^2 \end{aligned}$$

Which is the same answer as we got before. What I've seen oh, so many time is someone just using a 10^{-6} instead of the 10^{-12} that should be used here. This is a real trap, so beware that you don't fall into it!

Next, we want to express this in cm^2. This is really where the metric system makes its value clear. To go from one multiple of the base unit (μm) to another (cm) just requires shifting the decimal point. There are $10^4\ cm^2$ in each m^2. Thus, there are 10^8 μm^2 in each cm^2. So we can either multiply the answer found in square meters by 10^4 or divide the answer found in square microns by 10^8. Take your pick. (O.K., time for an error checking tool: At this point in the process, *pause*, take a breath, and check to see if your answer makes sense. Since square centimeters are *smaller* than square meters, the hair will occupy *more* square centimeters than square meters. Likewise, since square centimeters are *bigger* than square microns, the hair will occupy *less* square centimeters than square microns. A real source of error is to divide when you should multiply at this stage. Look at your answer and make sure that it makes sense, don't just plug blindly.) Using this, we get

$$A = 1.963\times 10^3\ \mu\text{m}^2 = 1.963\times 10^3\ \mu\text{m}^2 \times 10^{-8}\,\frac{cm^2}{\mu m^2} = 1.963\times 10^{-5}\,cm^2$$

Finally, to turn this into square inches, we need to use the conversion factor between inches and centimeters: $2.54\,cm = 1\,inch$. Once again, remember to square the conversion factor for the linear quantity when going to the area quantity. This gives

$$A = 1.963\times10^{-5}\,cm^2 = 1.963\times10^{-5}\,cm^2 \times \frac{1inch^2}{(2.54\,cm)^2}$$

$$= 1.963\times10^{-5}\,cm^2 \times \frac{1inch^2}{6.45\,cm^2} = 3.04\times10^{-6}\,inch^2$$

Once again, do a quick error-check: Since square inches are bigger than square centimeters, we expect the area to occupy a smaller fraction of a square inch than of a square centimeter. Our answer in square inches should be *smaller* than our answer in square centimeters. Indeed, it is. This, of course, doesn't prove that we got the right answer, but, had it worked out otherwise, it would have been a signal that we'd gotten the wrong answer.

4. **"Speed" is the distance an object travels divided by the time it takes to travel that distance. What are the dimensions of speed? What is the unit for speed within the *SI* system?**

As we will discuss in class, speed is distance traveled divided by the time it took to travel that distance. We will usually speak of *instantaneous* speed in this class. This is found by, mathematically, taking the time over which the distance is traveled to be indistinguishable from zero. This prevents us from having to take an average of speed over some time, in which the speed may have changed dramatically.

Since distance is a Length, the dimensions of speed are $\frac{\text{length}}{\text{time}}$.

5. **A man is walking with a speed of $2\frac{\text{meters}}{\text{second}}$. Express this speed in miles per hour.**

I gave this problem to you for two reasons: First, to give you practice converting a quantity from one set of units to another. Secondly, so that you would know the relationship between the units for speed (and velocity) that we will be using in this course almost exclusively, $\frac{\text{meters}}{\text{second}}$, and that which you are most accustomed to $\frac{\text{miles}}{\text{hour}}$. It would be worthwhile for you to memorize this relationship until you get an intuitive feel for $\frac{\text{meters}}{\text{second}}$. (I try very hard to use numbers in examples and on assignments that are realistic. Sometimes, mathematical convenience wins [or I make a careless mistake] and I use goofy numbers. I'll try to alert you when I've done this. Remember always that Physics is not intended to be a study of pure abstractions. It is supposed to aid us in understanding the world in which we actually live. Try to visualize the problems you are given and compare your visualization to the way you think things work. If you do this often enough, what you study in class will become visible in your daily life. In this example, the man is walking at quite a brisk pace—almost a jog. Try to picture it. Convert meters to feet [2 meters is just shy of 7 feet] if that will help you see it in your head.)

We'll need some intermediate conversion factors:

There are 5280 feet in one mile.

There are 12 inches in a foot.

There are 2.54 centimeters in an inch.

There are 100 centimeters in a meter.

There are 60 minutes in an hour.

There are 60 seconds in a minute.

That should do it. Let's do the lengths first.

$$2\,\frac{\text{meters}}{\text{second}}\times 100\,\frac{\text{cm}}{\text{m}}\times\frac{1}{2.54}\,\frac{\text{inch}}{\text{cm}}\times\frac{1}{12}\,\frac{\text{foot}}{\text{inch}}\times\frac{1}{5280}\,\frac{\text{mile}}{\text{foot}}$$

$$=\frac{200}{2.54\times 12\times 5280}\,\frac{\text{miles}}{\text{second}}=1.24\times 10^{-3}\,\frac{\text{miles}}{\text{second}}$$

So far, so good. Now we need to convert miles per second to miles per hour (remember that "per" is an English word that's easily translated into a mathematical operation).

$$1.24\times 10^{-3}\,\frac{\text{miles}}{\text{second}}\times 60\,\frac{\text{second}}{\text{minute}}\times 60\,\frac{\text{minutes}}{\text{hour}}=3600\times 1.24\times 10^{-3}\,\frac{\text{miles}}{\text{hour}}=4.46\,\frac{\text{miles}}{\text{hour}}$$

So, $2\,\frac{\text{meters}}{\text{second}}$ is the same as $4.46\,\frac{\text{miles}}{\text{hour}}$. Does this match what you visualized?

6. **A very lazy snail is sliding along at a speed of $1\,\frac{\text{furlong}}{\text{fortnight}}$. Express this speed in meters per second.**

I decided to have some fun with this one. The previous problem deals with a conversion that you'll have occasion to do fairly often. I doubt if you'll *ever* need to do this one! But it's still a good exercise. We proceed exactly as before. We'll need a few more conversion factors:

There are 14 days in a fortnight.

There are 24 hours in a day.

There are 8 furlongs in a mile.

Let's use the result from the previous problem: $2\,\frac{\text{meters}}{\text{second}}=4.46\,\frac{\text{miles}}{\text{hour}}$, so $1\,\frac{\text{mile}}{\text{hour}}=\frac{2}{4.46}\,\frac{\text{meters}}{\text{second}}=.448\,\frac{\text{meters}}{\text{second}}$ (all I did there was divide both sides by 4.46). We can write this as $.448\,\frac{\frac{\text{meters}}{\text{second}}}{\frac{\text{mile}}{\text{hour}}}$—in words, this reads ".448 meters per second per mile per hour.” Now, let’s turn furlongs per fortnight into miles per hour using the conversion factors above:

$$1\,\frac{\text{furlong}}{\text{fortnight}}\times\frac{1}{14}\,\frac{\text{fortnight}}{\text{day}}\times\frac{1}{24}\,\frac{\text{day}}{\text{hour}}=\frac{1}{14\times 24}\,\frac{\text{furlong}}{\text{hour}}=2.976\times10^{-3}\,\frac{\text{furlong}}{\text{hour}}$$

Now, before we go further, this is a good time to do a little quick-and-dirty error checking: Whenever you do a calculation, it’s a good idea to pause every now and again to make sure the answer you’re getting makes sense. Make sure you haven’t really gummed something up. We’ve turned furlongs per fortnight into furlongs per hour. How many hours in a fortnight? Well, there are about 25 hours in a day and about 10 days in a fortnight (remember, we’re being quick-and-dirty here—the idea is to do it really quickly just to get an *idea* of the reasonableness of the answer, not to get the answer itself, so choose numbers that are easy to calculate with). That’s about 250 hours and 1/250=4/1000. So getting 3/1000 of a fortnight in one hour seems alright. We can move on with confidence! Now, we do one quick step: Turn the furlongs into miles by dividing by 8. $2.976\times10^{-3}\,\frac{\text{furlong}}{\text{hour}}\times\frac{1}{8}\,\frac{\text{mile}}{\text{furlong}}=3.72\times10^{-4}\,\frac{\text{mile}}{\text{hour}}$. Finally, we just use the result from the previous problem to turn miles per hour into meters per second: $3.72\times10^{-4}\,\frac{\text{mile}}{\text{hour}}\times .448\,\frac{\frac{\text{meters}}{\text{second}}}{\frac{\text{mile}}{\text{hour}}}=1.67\times10^{-4}\,\frac{\text{meters}}{\text{second}}$. So, by comparing this to problem #2, we see that our lazy snail meanders along at about three hair-widths every second.

Second part: Position and motion

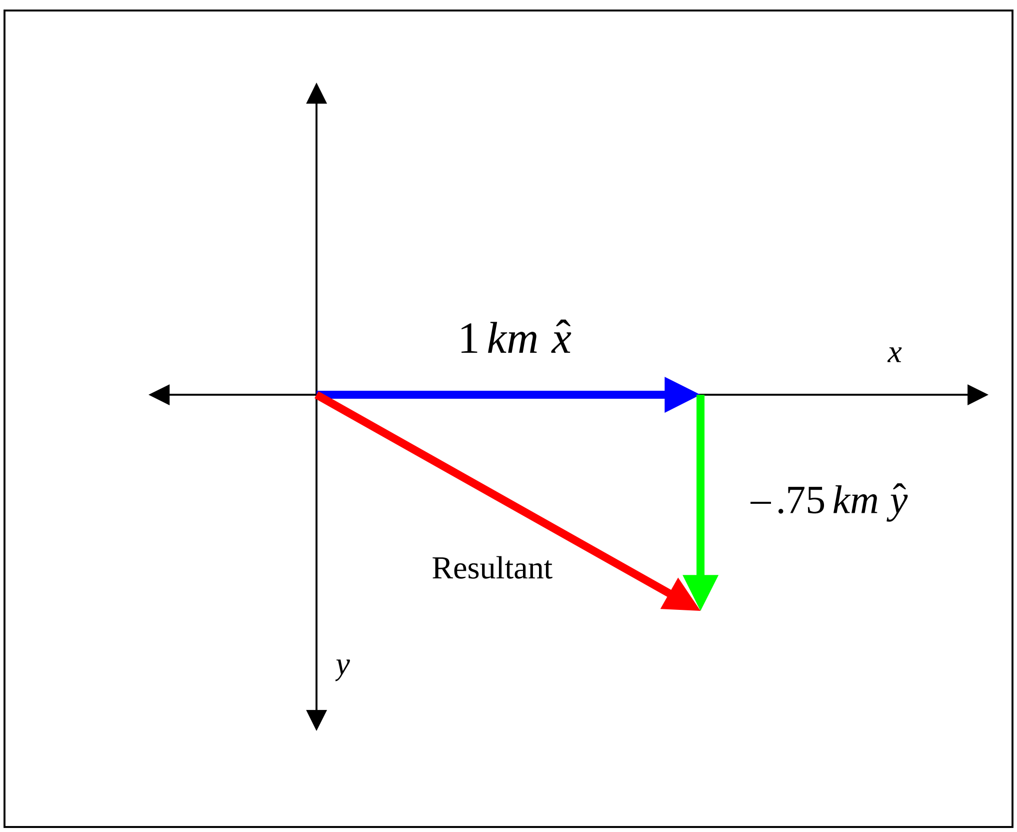
7. **A woman goes for a walk. She begins at the origin of a Cartesian coordinate system. After walking in a straight line for one kilometer, she turns right and walks perpendicular to her original path for ¾ of a kilometer.**

   a. **What are her coordinates at the end of this walk? Make whatever assumption you deem necessary to answer this. Explain your assumptions.**

   b. **What total distance did the woman walk?**

   c. **What is the woman's total displacement at the end of her trip (measured from her beginning point)?**

We are told that the woman begins at the origin of the coordinate system. We are not told anything about the orientation of the coordinate system. When you're not told something like that, you might as well assume whatever will make you life easiest—there are plenty of things out there to make it tough (that's *my* job!), so take the breaks when you get them. We assume that the woman begins walking along one of the Cartesian axes—that is, either along the x or the y axis. I'll pick the x axis. So, she walks in the positive x direction for one km and then turns right, which points her in the negative y direction. This is shown below:



So her coordinates are just the *x* and *y* values of her position. We write this (1 *km*,-.75 *km*).

The total distance she walked is just the amount that she walked in the *x* direction plus the amount that she walked in the *y* direction. Be careful not to include the negative sign for the *y* displacement! The minus sign indicates the orientation of her walking. It doesn't mean that she somehow walked a negative distance. So, she walked a total of 1.75 *km*.

Her total displacement is another story: This is how far she wound up from where she started. The fact that she chose an inefficient path doesn't change anything. She wound up at (1 *km*, -.75 *km*). We use the Pythagorean theorem to find the total distance (the hypotenuse of the triangle formed by her displacement in the *x* direction and her displacement in the *y* direction): $D=\sqrt{1^2+.75^2}=\sqrt{1+.5625}=1.25\,km$ (Note my sloppiness: I really should have included the units in every single step in this. I got a little lazy and just stuck them in at the end. I'll let you get away with this provided that you don't make a mistake! Getting the wrong units at the end will cost you significant partial credit. Failing to put units in at all will cause you to lose all credit.)

8. **Beginning in the sixteenth century (according to the best estimates), ships determined their speed, relative to the water, by throwing a "chip log" over the side. The log was attached to the ship by a rope into which knots were tied every 47.25 feet. The rope was allowed to unroll off a spool as the ship moved forward. The number of knots that came off the spool in a 30 second period was counted.**
   a. **If one "knot" came off the spool in 30 seconds, the ship was said to be traveling at a speed of 1 knot. How many "statute" (i.e., land-based) miles per hour is 1 knot? (1 statute mile = 5280 feet)**
   b. **One Nautical Mile is the distance a ship travels in one hour if it travels at a speed of one knot for one hour. It is *also* the length of one minute of arc along the Earth's equator. Using this, what is the circumference of the Earth in nautical miles?**
   c. **Using the result from (a) and (b), what is the circumference of the Earth in statute miles?**

Here's one that looks a *lot* tougher than it actually is. A primary goal of this course is to train you to think mathematically. This means starting with a description of a system, you must learn to render that description into a mathematical form. So I've given you a long-winded description (containing some cool history) with the challenge of extracting some relatively straightforward features in mathematical form.

From the question, we know that the speed of the ship is $v=\frac{\Delta s}{\Delta t}=\frac{47.25\,ft}{30\,s}$. But this will give us the speed in feet per second. We want this in miles per hour. So let's turn the feet into miles and the seconds into hours. It's best to do this in steps until you get good at it. $v=\frac{\Delta s}{\Delta t}=\frac{47.25\,ft}{30\,s}=\frac{47.25\,ft}{30\,s}\times\frac{1mile}{5280\,ft}=\frac{8.949\times10^{-3}mile}{30s}$. Time for a quick check—the most common error to make at this point would be to multiply by the 5280 instead of dividing by it. Note that this would have given us units of $\frac{feet^2}{mile\cdot second}$, which is clearly incorrect. Another check is just the *size* of the answer: We expect the number of miles traversed in a certain amount of time to be significantly less than the number of feet traversed in that same amount of time, so we'd better get a pretty small number in the numerator, which we do.

Next, let's turn our 30 seconds into hours $v=\frac{\Delta s}{\Delta t}=\frac{8.949\times10^{-3}mile}{30s}\times\frac{3600\,s}{1hour}=1.074\frac{mile}{hour}$.

An important point must be made here: It turns out that the standard sandglass used to measure the 30 second time interval on ships was incorrect. Using precise clocks (which were not available until the late 18th century, by which time the traditions were well-entrenched), it was determined that the standard glass actually measured 28 seconds. Repeating the above calculation, we get $v=\frac{\Delta s}{\Delta t}=\frac{8.949\times10^{-3}mile}{28s}\times\frac{3600\,s}{1hour}=1.15\frac{mile}{hour}$. This is the standard definition of the "knot" that you will find listed for measuring speeds by sailors and airplane pilots.

Now, to determine the circumference of the Earth, very little calculation is needed. Since a nautical mile is the length of one minute of arc and there are 60 minutes of arc in one degree and 360 degrees in the full circumference, all we need to do is multiply: $c=1\frac{mile}{arc-minute}\times60\frac{minutes}{degree}\times360\frac{degrees}{circle}=21600\,miles$.

Finally, since in one hour a ship traveling at one knot travels 1.074 statute miles (we'll stick with the value we calculated since this was a problem in calculation, not in looking-up [oh, by the way, if you used a looked-up value instead of a calculated value on this problem, you'll get a 0 on it; if you *augmented* your calculated answer with a looked-up one, that's just fine, you won't be penalized]) and *also* travels 1 nautical mile, it is clear that the conversion between the two kinds of mile is $1mile(nautical)=1.074\,mile(statute)$. Thus the circumference of the Earth is $c=21600\,miles(nautical)\times\frac{1.074\,mile(statute)}{1mile(nautical)}=23198\,mile(statute)$. (Once again, the *true* answer would have used a conversion factor of 1.15 instead of 1.074. This would have yielded

$$c=21600\,miles(nautical)\times\frac{1.15\,mile(statute)}{1mile(nautical)}=24840mile(statute).)$$

9. **Consider a Cartesian coordinate system. A man goes for a walk beginning at a point which has coordinates $(-25.4\ \mathrm{m}, 38.2\ \mathrm{m})$—let's call the two directions x and y for the first and second number in the pair, respectively. He walks in the positive x direction for 22 meters and then walks in the positive y direction for 14 meters.**

   a. **What are his coordinates at the end of the walk?**

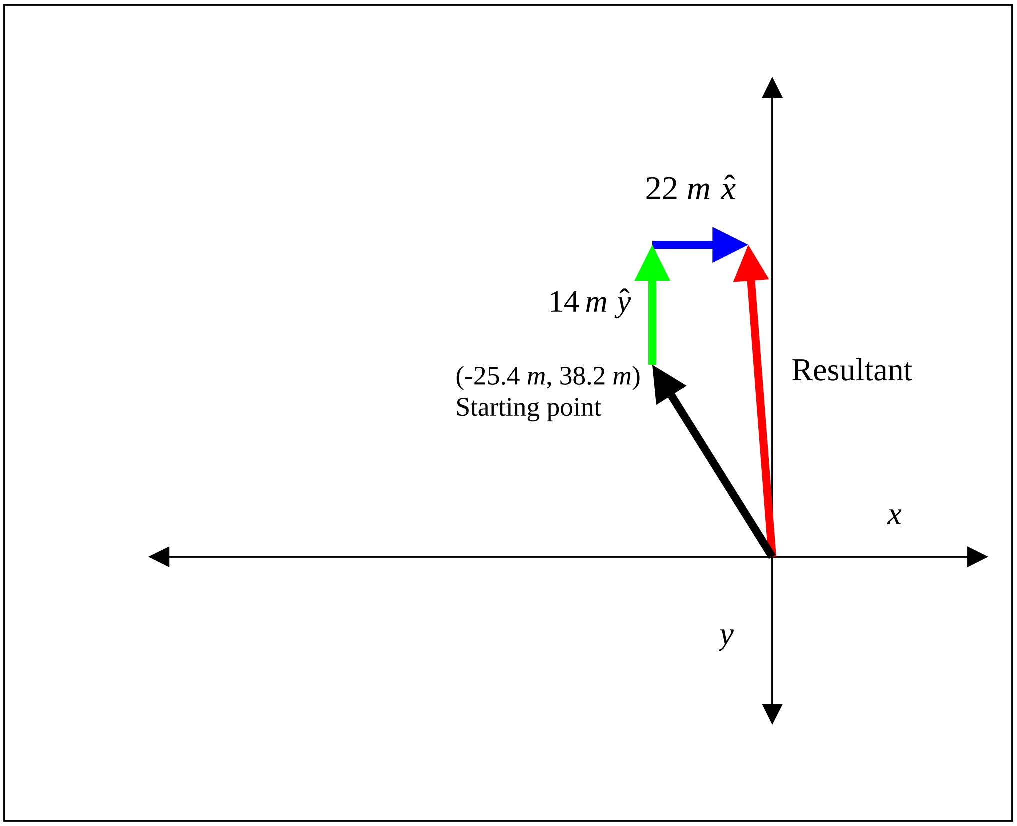
   b. **What total distance did he walk?**

   c. **What is his total distance from the origin of the coordinate system?**

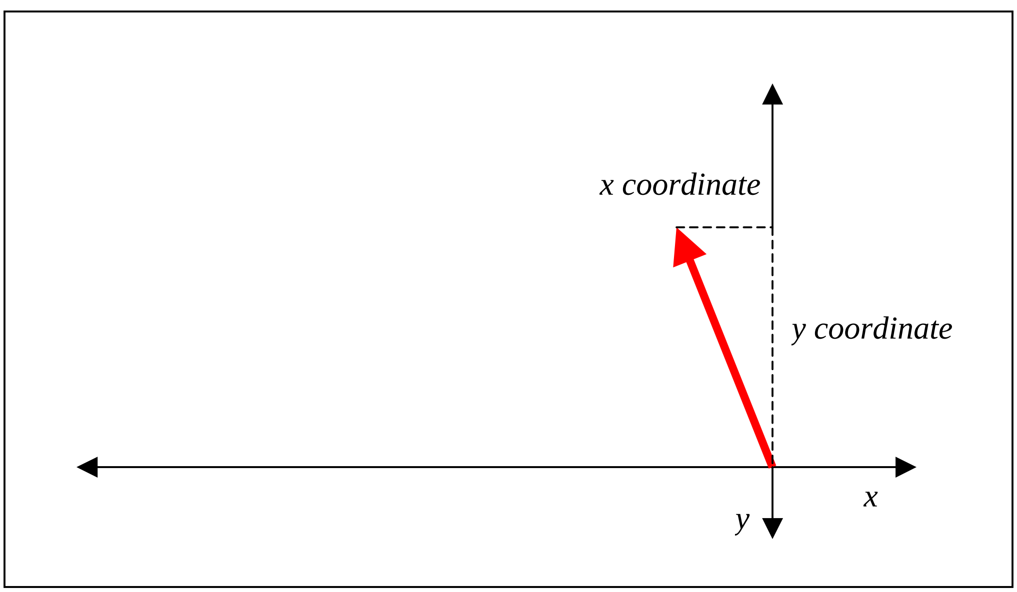
Now, things are getting a bit more difficult. Not only do we not have the freedom of selecting our coordinate system's orientation, we aren't even starting at the origin! Fortunately, the first question can be handled in a very straightforward manner: To find the coordinates at the end, just add the coordinates at the beginning to the distances he walked in the *x* and *y* directions. His coordinates at the end of the walk are (-25.4 *m*+22 *m*, 38.2 *m*+14 *m*) or (-3.4 *m*, 52.2 *m*).

As with the previous problem, the total distance that he walked is just the amount that he walked in the *x* direction added to the total distance he walked in the *y* direction: 22 *m* + 14 *m*=36 *m*. Note that he *started* at (-25.4 *m*, 38.2 *m*). Do not add those coordinates to the distance that he walked: We have no idea how he got to the starting point. There's no reason to assume that he started at the origin. Nor is there any reason to assume that he got there by walking in straight lines.

Finally, we need to figure out his total distance from the origin. This is shown below:



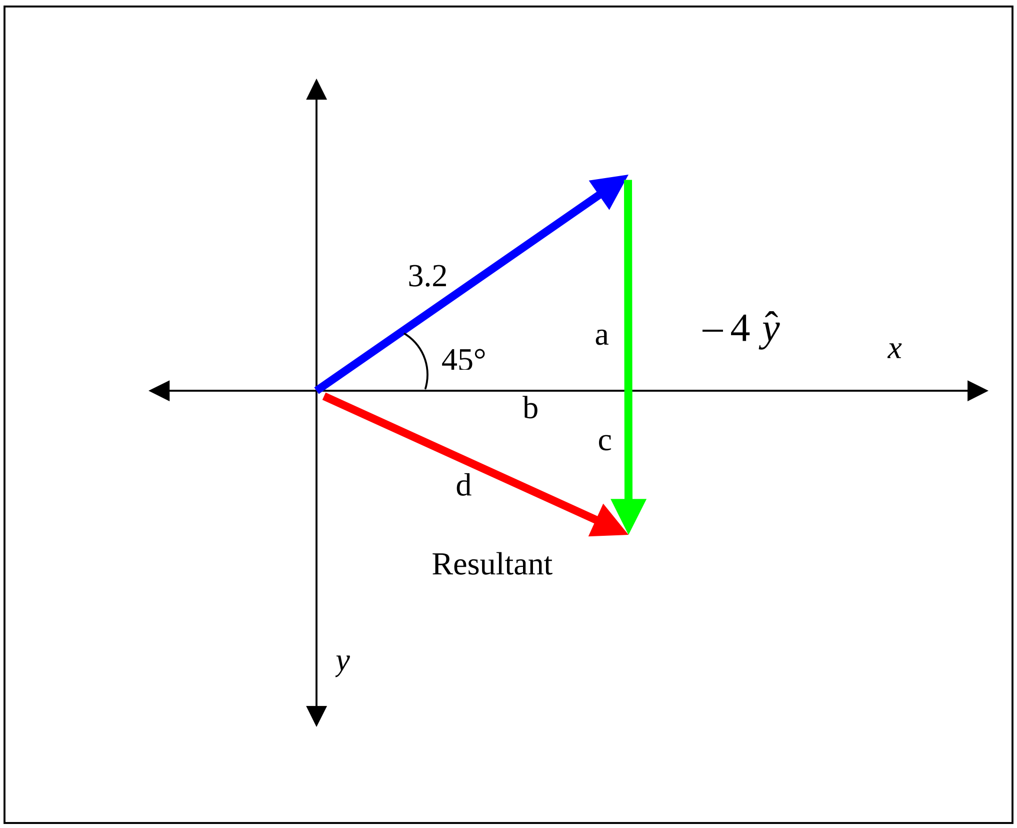
At first glance, trying to find the length of the resultant seems *very* daunting! But note that we know the coordinates of the final position. Big deal, what does that buy us? Well, recall that the x coordinate is the distance we are from the y axis and the y coordinate is the distance we are from the x axis. Read that over a couple of times and then look at the picture below:



Notice that the x and y coordinates of the final position are precisely the lengths of two legs of a right triangle (dotted lines) and the final displacement vector is the hypotenuse of that triangle! Well, now finding the total displacement is easy: Just use Pythagoras's handy theorem once again, plugging in the coordinates for the endpoint of the trip. This gives $D = \sqrt{(3.4\,m)^2 + (52.2\,m)^2} = 52.3\,m$.

10. **A vector oriented 45° relative to the x axis is added to a vector oriented parallel to the y axis. The first vector has a length of 3.2. The second vector has a length of –4 (i.e., it's pointed in the negative y direction). What is the length of the "resultant," i.e., the vector that results from the addition of the two?**

When adding vectors, the order in which we do the addition doesn't matter. Think about that for a second: If I walk a mile north and then a half mile east I get to the same place as I would had I walked a half mile east and then a mile north, don't I? So you can do this in any order that's convenient. To get a visual representation of a vector addition, draw the vectors in head-to-tail. At the end, draw the resultant vector by drawing a vector from the tail of the first vector to the head of the last vector. (In this case, there are only two. In future problems, there may be a large number added together. The method is the same: Just chain them together, head-to-tail.)



Now, we can do this problem two different ways: We can either use geometry and trigonometry to find the solution, or we can just do it algebraically with a bit of trig. I'll do it both ways. Let's start with the geometrical approach.

Looking at the figure, we see that the vector in the negative *y* direction ends up below the axis. One concern we always have when using a picture to help with a solution is that the picture will not be accurate and we'll be led astray by it. Be careful! In this case, you know that the *y* vector will dip below the axis because the 45° vector is only 3.2 long (note that there are no units in this problem). The 45° vector forms the hypotenuse of a right triangle with portions of the *x* axis and *y* axis as its legs. Since the hypotenuse is always the longest leg of a right triangle, the *y* leg must be shorter than 3.2. So *part* of the *y* vector must dip below the *x* axis.

To figure out how much of the *y* vector is above the axis, we use a little trigonometry. We use SOHCAHTOA to determine that length. SOHCAHTOA tells us that the length can be found from the sine of 45°. I've labeled this as "a" on the figure. We have $a = 3.2\sin(45°) = 2.26$.

Notice that there is a triangle formed by the *x* axis and the piece of the *y* vector that dips below the *x* axis and the resultant. I've labeled these three sides b, c, and d respectively. We can find c by subtracting a from the total length of the *y* vector. $c = 4 - 2.26 = 1.74$. Also, notice that b can be found from SOHCAHTOA as well: $b = 3.2\cos(45°) = 2.26$. Now, here's a possible source of confusion: The fact that I got the same number for b as for a is just a result of the fact that my angle is 45°. When the angle is 45°, the sine and the cosine are the same. This is not the case for other angles!

We now have b and c, so we can find d using the theorem of Pythagoras. $d = \sqrt{2.26^2 + 1.74^2} = 2.85$. This is the length of our resultant.

The second way of doing this is to use algebra. Recall that we can write down a vector several different ways. One way is the way I did it in this problem—by expressing a length and an angle in some coordinate system. Another way is to express the components of the vector. To go from the one way to the other, one has to use a bit of trigonometry. We can express the vector that's 3.2 long, 45° relative to the *x* axis as $2.26\hat{x} + 2.26\hat{y}$. (Note that I used the trig results found above to write these components.) The second vector can be written $-4\hat{y}$. Now, to find the components of the resultant, I just add the $\hat{x}$ pieces together and the $\hat{y}$ pieces together. Let's call the resultant vector $\vec{d}$. The math is very simple $\vec{d} = 2.26\hat{x} + (2.26 - 4)\hat{y} = 2.26\hat{x} - 1.74\hat{y}$. To find the length of any vector, square the components, add them together, and take the square-root of the sum. $d = \sqrt{2.26^2 + 1.74^2} = 2.85$. Exactly as before.

Although it doesn't always save one from doing some geometry and some trigonometry, using the component method is usually much more straightforward. The trick is never to lose sight of the underlying geometry, even if you don't need to manipulate it directly.