PSD Chip Calculations

Energy Conversions

Erad	Energy of incident radiation (MeV)
evis	Energy of visible photon radiation (eV)
εcon	Conversion efficiency
εcoll	Visible light collection efficiency
εq	Photocathode quantum efficiency
εsplit	Split signal efficiency

$$LY = \frac{10^6}{\varepsilon vis} \cdot \varepsilon con$$

$$kcn = LY \cdot \varepsilon coll \cdot \varepsilon q$$

$$Ne = Erad \cdot kcn$$

$$evis = 3$$

$$\varepsilon con = \begin{cases} 0.17 & \text{for CsI(Tl)} \\ 0.045 & \text{for Liquid Scintillator} \end{cases}$$

$$\varepsilon coll = 0.8$$

$$\varepsilon q = 0.25$$

$$\varepsilon split = 0.5$$

Transresistive Gain Calculation

$$Ar_{GAIN} = \frac{V_{IN,\max}}{E_{\max} \cdot kcn \cdot q \cdot \max\left(f\left(t\right)\right)}$$

- Ar_{gain} is the Transresistive Gain following the energy conversion of an incoming charge pulse.
- $V_{IN,max}$ is the maximum voltage allowed at the input of the chip.
- E_{max} is the maximum energy value that will produce the maximum voltage.

Pulse Model

Multi-Exponential (with rise and fall times) (Normalized)

Pulse Creation Equations

$$f(t) = \frac{1}{\sum_{i=1}^{n} A_i} \cdot \sum_{i=1}^{n} \left(\frac{A_1}{\tau_{F,i} - \tau_{R,i}} \cdot \left(e^{-\frac{t}{\tau_{F,i}}} - e^{-\frac{t}{\tau_{R,i}}} \right) \right) \quad \text{for } n \text{ exponentials}$$

$$\int_0^{\infty} f(t) = 1$$

$$f_V(t) = \frac{f(t)}{\max(f(t))}$$

$$\max(f_V(t)) = 1$$

Pulse Integration Equations

$$F(t_1, t_2) = \int_{t_1}^{t_2} f(t)dt = \frac{1}{\sum_{i=1}^{n} A_i} \cdot \sum_{i=1}^{n} \left(\frac{A_i}{\tau_{F,i} - \tau_{R,i}} \cdot \left(\tau_{F,i} \cdot \left(e^{-\frac{t_1}{\tau_{F,i}}} - e^{-\frac{t_2}{\tau_{F,i}}} \right) - \tau_{R,i} \cdot \left(e^{-\frac{t_1}{\tau_{R,i}}} - e^{-\frac{t_2}{\tau_{R,i}}} \right) \right) \right)$$

$$F_V(t_1, t_2) = \frac{1}{\tau_{INT}} \cdot \frac{F(t_1, t_2)}{\max(f(t))}$$

Noise Sources

- ***Poisson*** – noise due to random arrival of discrete electrons
- ***Electronics Noise***
  - ***Jitter*** – noise created by an uncertainty in the integration start time and in the width of integration period
  - ***RI*** – thermal noise from the integrating resistor sampled onto the integrating capacitor
  - ***OTA*** – thermal noise of the op amp sampled onto the integrating capacitor
  - ***OTA (+)*** – continuous additive input-referred thermal noise of the op amp
  - ***1/f*** – 1/f noise of the op amp sampled onto the integrating capacitor
  - ***1/f (+)*** – continuous additive input-referred 1/f noise of the op amp
- ***ADC*** – quantization noise of a 12-bit converter

Poisson Noise

$$k_{OUT} = \frac{q \cdot Ar_{GAIN}}{\tau_{INT}}$$

$$\sigma_p^{\,2} = k_{OUT} \cdot |V_{OUT}|$$

- k_{OUT} represents the gain from incoming charge packet to voltage output
- σ_p^2 is the variance of the Poisson noise at the output of the integrator.

Jitter Noise

$$VOF_i = \frac{A}{\tau_{Fi}-\tau_{Ri}} \cdot \tau_{Fi} \cdot e^{-\frac{Ti}{\tau_{Fi}}} \cdot \left(1-e^{-\frac{T}{\tau_F}}\right)$$

$$VOR_i = -\frac{A}{\tau_{Fi}-\tau_{Ri}} \cdot \tau_{Ri} \cdot e^{-\frac{Ti}{\tau_{Ri}}} \cdot \left(1-e^{-\frac{T}{\tau_{Ri}}}\right)$$

$$c_{i,Ti} = -\left(\frac{VOF_i}{\tau_{Fi}} + \frac{VOR_i}{\tau_{Ri}}\right)$$

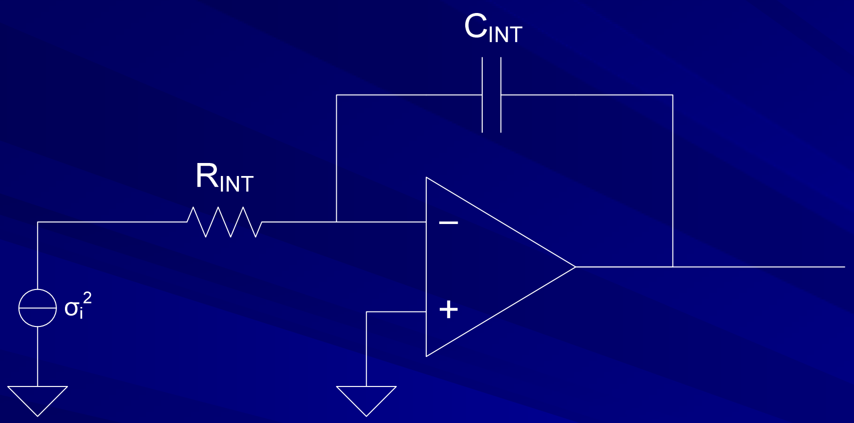
$$c_{i,T} = \frac{VOF_i}{\tau_{Fi}} \frac{e^{-T/\tau_{Fi}}}{1-e^{-T/\tau_{Fi}}} + \frac{VOR_i}{\tau_{Ri}} \frac{e^{-T/\tau_{Ri}}}{1-e^{-T/\tau_{Ri}}}$$

where i = 1, 2, …, n
for n exponentials

$$\sigma_j^{\ 2} = \left(\sum_{i=1}^{n} c_{i,Ti}\right)^2 \sigma_{Ti}^{\ 2} + \left(\sum_{i=1}^{n} c_{i,T}\right)^2 \sigma_T^{\ 2}$$

- VOF and VOR are the separate voltages at the output for the falling and rising exponentials.
- $C_{i,Ti}$ and $C_{i,T}$ are the constants for *n* exponentials involved in the calculation of variance at the output.
- σ_j^2 is the variance at the output due to jitter in the starting integration, Ti, and integration period, T at the input.

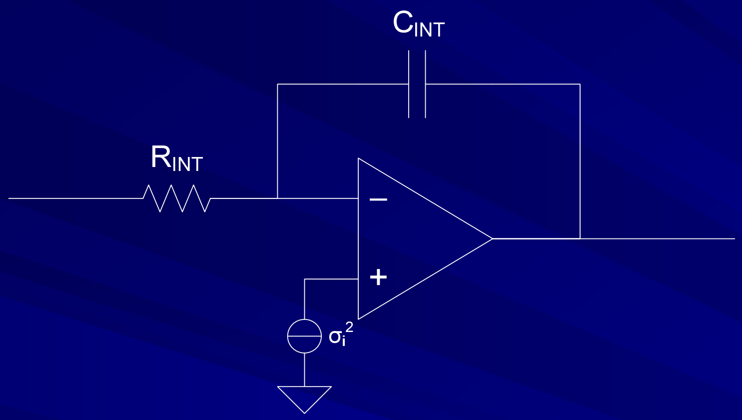
Integrating Resistor Thermal Noise (Sampled)



$$\sigma_{RI,t}^{2} = 4\frac{kT}{C_{INT}}\frac{T}{\tau_{INT}}$$

- $\sigma_{RI,t}^2$ is the variance sampled onto the integrating capacitor due to thermal noise in the integrating resistor.

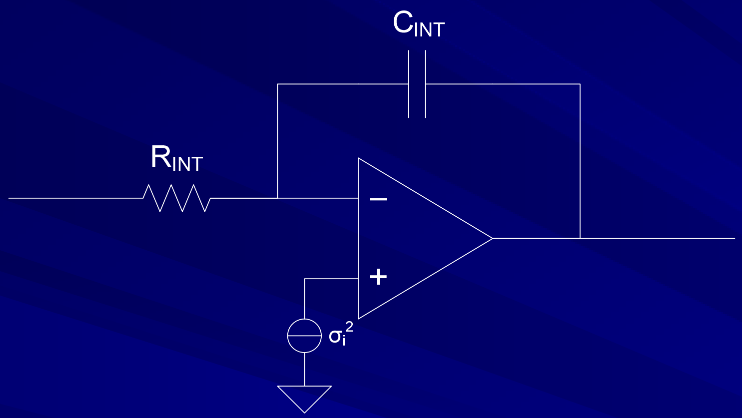
OTA Thermal Noise (Sampled)



$$\sigma_{OTA,t}^{\;2} = \sigma_{RI,t}^{\;2} \cdot \frac{RN}{R_{INT}}$$

- RN is the equivalent thermal resistance of the OTA.
- $\sigma_{OTA,t}^{2}$ is the variance sampled onto the integrating capacitor due to thermal noise in the OTA.

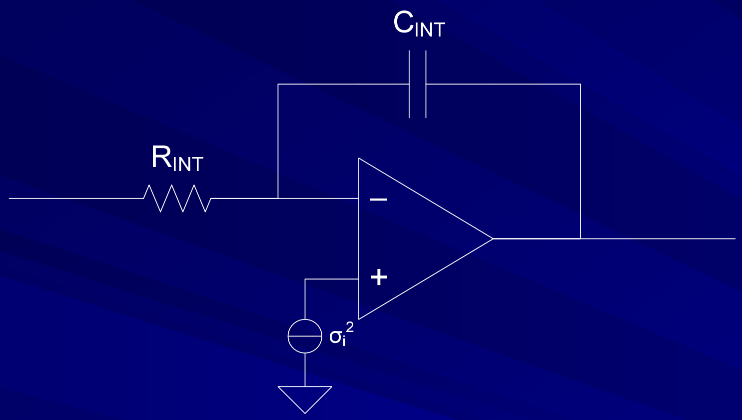
OTA Thermal Noise (Continuous)



$$\sigma_{OTA+,t}{}^{2} = 4 \cdot kT \cdot RN \cdot BW$$

- RN is the equivalent thermal resistance of the OTA.
- BW is the close-loop bandwidth of the OTA.
- $\sigma_{OTA,t}{}^{2}$ is the continuous-time variance at the output due to thermal noise in the OTA.

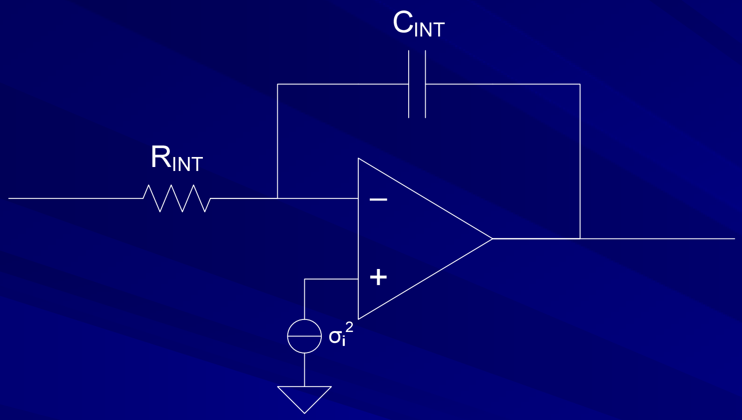
OTA 1/f Noise (Sampled)



$$\sigma_{OTA,f}^{\;2} = Kf \cdot \left[\ln\left(2 \cdot T_{cal} \cdot Fs\right)^{0.62}\right]^2$$

- T_{cal} is the time span between calibrations of the output voltage.
- Fs is the sampling frequency, or twice the bandwidth of the voltage at the output.
- Kf is the fitted 1/f constant that models the 1/f noise in the OTA.
- $\sigma_{OTA,f}^2$ is the variance sampled onto the integrating capacitor due to 1/f noise in the OTA.

OTA 1/f Noise (Continuous)



$$\sigma_{OTA+,f}^{\;2} = \sigma_{OTA,f}^{\;2} \cdot \frac{T}{\tau_{INT}}$$

- $\sigma_{OTA+,f}^{2}$ is the continuous-time variance at the output due to 1/f noise in the OTA.

ADC Quantization Noise

$$Q_{bin} = \frac{VO_{\max}}{2^{ADC_{bits}}}$$

$$\sigma_{ADC}{}^{2} = \frac{Q_{bin}{}^{2}}{12}$$

- Q_{bin} is the quantization bin size of an ADC with ADC_{bits} of resolution.
- $\sigma_{ADC}{}^{2}$ is the variance of the ADC at the output.

Variance and SNR at the output

$$\sigma_{TOTAL}{}^2 = \sigma_p{}^2 + \sigma_j{}^2 + \sigma_{RI,t}{}^2 + \sigma_{OTA,t}{}^2 + \sigma_{OTA+,t}{}^2 + \sigma_{OTA,f}{}^2 + \sigma_{OTA+,f}{}^2$$

$$SNR = \frac{VOUT}{\sigma_{TOTAL}}$$

- Since each noise variance at the output is independent of each other, the total variance at the output is simply the sum of the variances.
- SNR = Signal to Noise Ratio

Analytical Predictions of Variance of Angular PSD Plots

$$\mathrm{var}(\theta) = \frac{\sin^2 2\theta}{4} \cdot \left[\frac{1}{SNR_A{}^2} + \frac{1}{SNR_B{}^2} \right]$$

$$FOM = \frac{\left|\theta_1 - \theta_0\right|}{\sqrt{\mathrm{var}(\theta_1) + \mathrm{var}(\theta_0)}}$$

- Variance of angular PSD plot depends on the signal-to-noise ratio of the A and B integrators.
- Small signal-to-noise ratios, which correspond to low-energy particles, results in a larger variance in angle which is consistent with simulation.
- Figure of merit (FOM) is computed as the difference between the means divided by the square root of the sum of the variances.