My Beautiful Beamer Presentation

it's really gorgeous

Karl Gauss

Department of Mathematics and Statistics
Southern Illinois University Edwardsville

Conference on Important Mathematical Objects

Outline

Outline

Definition of a Minimal Surface

Definition

A minimal surface is a 2-dimensional surface in $\mathbb{R}^3$ with mean curvature $H \equiv 0$.

Where does the name minimal come from?

Let $F : U \subset \mathbb{C} \to \mathbb{R}^3$ parameterize a minimal surface; let $d : U \to \mathbb{R}$ be smooth with compact support. Define a deformation of M by $F_\varepsilon : p \mapsto F(p) + \varepsilon d(p) N(p)$.

$$\frac{d}{d\varepsilon} Area(F_\varepsilon(U))\bigg|_{\varepsilon=0} = 0 \iff H \equiv 0$$

Thus, "minimal surfaces" may really only be critical points for the area functional (but the name has stuck).

Definition of a Minimal Surface

Definition

A minimal surface is a 2-dimensional surface in $\mathbb{R}^3$ with mean curvature $H \equiv 0$.

Where does the name minimal come from?

Let $F : U \subset \mathbb{C} \to \mathbb{R}^3$ parameterize a minimal surface; let $d : U \to \mathbb{R}$ be smooth with compact support. Define a deformation of M by $F_\varepsilon : p \mapsto F(p) + \varepsilon d(p) N(p)$.

$$\frac{d}{d\varepsilon} Area(F_\varepsilon(U))\Big|_{\varepsilon=0} = 0 \iff H \equiv 0$$

Thus, “minimal surfaces” may really only be critical points for the area functional (but the name has stuck).

Definition of Triply Periodic Minimal Surface

Definition

A triply periodic minimal surface M is a minimal surface in $\mathbb{R}^3$ that is invariant under the action of a lattice Λ. The quotient surface $M/\Lambda \subset \mathbb{R}^3/\Lambda$ is compact and minimal.

Physical scientists are interested in these surfaces:

- Interface in polymers
- Physical assembly during chemical reactions
- Microcelluar membrane structures

Definition of Triply Periodic Minimal Surface

Definition

A triply periodic minimal surface M is a minimal surface in $\mathbb{R}^3$ that is invariant under the action of a lattice Λ. The quotient surface $M/\Lambda \subset \mathbb{R}^3/\Lambda$ is compact and minimal.

Physical scientists are interested in these surfaces:

- Interface in polymers
- Physical assembly during chemical reactions
- Microcelluar membrane structures

Classification of TPMS

Rough classification by the genus of M/Λ:

Theorem

(Meeks, 1975) Let M be a triply periodic minimal surface of genus g. The Gauss map of M/Λ is a conformal branched covering map of the sphere of degree $g-1$.

Proof.

Since M is minimal, G is holomorphic (Weierstraß). Then M/Λ is a conformal branched cover of S^2. By Gauss-Bonnet:

$$-\mathit{degree}(G)4\pi = -\int |K| dA = \int K dA = 2\pi\chi(M) = 4\pi(1-g)$$

□

Corollary

The smallest possible genus of M/Λ is 3.

Classification of TPMS

Rough classification by the genus of M/Λ:

Theorem

(Meeks, 1975) Let M be a triply periodic minimal surface of genus g. The Gauss map of M/Λ is a conformal branched covering map of the sphere of degree $g-1$.

Proof.

Since M is minimal, G is holomorphic (Weierstraß). Then M/Λ is a conformal branched cover of S^2. By Gauss-Bonnet:

$$-\,degree(G)4\pi = -\int |K| dA = \int K dA = 2\pi\chi(M) = 4\pi(1-g)$$

□

Corollary

The smallest possible genus of M/Λ is 3.

Classification of TPMS

Rough classification by the genus of M/Λ:

Theorem

(Meeks, 1975) Let M be a triply periodic minimal surface of genus g. The Gauss map of M/Λ is a conformal branched covering map of the sphere of degree $g-1$.

Proof.

Since M is minimal, G is holomorphic (Weierstraß). Then M/Λ is a conformal branched cover of S^2. By Gauss-Bonnet:

$$-\,degree(G)4\pi = -\int |K| dA = \int K dA = 2\pi\chi(M) = 4\pi(1-g)$$

□

Corollary

The smallest possible genus of M/Λ is 3.

Classification of TPMS

Rough classification by the genus of M/Λ:

Theorem

(Meeks, 1975) Let M be a triply periodic minimal surface of genus g. The Gauss map of M/Λ is a conformal branched covering map of the sphere of degree $g-1$.

Proof.

Since M is minimal, G is holomorphic (Weierstraß). Then M/Λ is a conformal branched cover of S^2. By Gauss-Bonnet:

$$-\,degree(G)4\pi = -\int |K| dA = \int K dA = 2\pi\chi(M) = 4\pi(1-g)$$

□

Corollary

The smallest possible genus of M/Λ is 3.

Classification of TPMS

Rough classification by the genus of M/Λ:

Theorem

(Meeks, 1975) Let M be a triply periodic minimal surface of genus g. The Gauss map of M/Λ is a conformal branched covering map of the sphere of degree $g-1$.

Proof.

Since M is minimal, G is holomorphic (Weierstraß). Then M/Λ is a conformal branched cover of S^2. By Gauss-Bonnet:

$$-degree(G)4\pi = -\int |K| dA = \int K dA = 2\pi\chi(M) = 4\pi(1-g)$$

□

Corollary

The smallest possible genus of M/Λ is 3.

Classification of TPMS

Rough classification by the genus of M/Λ:

Theorem

(Meeks, 1975) Let M be a triply periodic minimal surface of genus g. The Gauss map of M/Λ is a conformal branched covering map of the sphere of degree $g-1$.

Proof.

Since M is minimal, G is holomorphic (Weierstraß). Then M/Λ is a conformal branched cover of S^2. By Gauss-Bonnet:

$$-degree(G)4\pi = -\int |K| dA = \int K dA = 2\pi\chi(M) = 4\pi(1-g)$$

□

Corollary

The smallest possible genus of M/Λ is 3.

Other classifications?

Many triply periodic surfaces are known to come in a continuous family (or deformation).

Theorem

(Meeks, 1975) There is a five-dimensional continuous family of embedded triply periodic minimal surfaces of genus 3.

▸ Picture

All proven examples of genus 3 triply periodic surfaces are in the Meeks' family, with two exceptions, the gyroid and the lidinoid.

Other classifications?

Many triply periodic surfaces are known to come in a continuous family (or deformation).

Theorem

(Meeks, 1975) There is a five-dimensional continuous family of embedded triply periodic minimal surfaces of genus 3.

▸ Picture

All proven examples of genus 3 triply periodic surfaces are in the Meeks' family, with two exceptions, the gyroid and the lidinoid.

The Gyroid

- Schoen, 1970
- Triply periodic surface
- Contains no straight lines or planar symmetry curves

Outline

Philosophy of the Problem

From $H \equiv 0$ to Complex Analysis

Using Weierstraß Representation construct surfaces by finding a Riemann surface X, a meromorphic function G on X, and a holomorphic 1-form dh on the X so that:

- The period problem is solved
- Certain mild compatibility conditions are satisfied

From Complex Analysis to Euclidean Polygons

The period problem is typically hard. Using flat structures, transfer the period problem to one involving Euclidean polygons and compute explicitly (algebraically!) the periods. To achieve this we:

- Assume (fix) some symmetries of the surface to reduce the number of parameters (and the number of conditions)
- Find a suitable class of polygons to study

Philosophy of the Problem

From $H \equiv 0$ to Complex Analysis

Using Weierstraß Representation construct surfaces by finding a Riemann surface X, a meromorphic function G on X, and a holomorphic 1-form dh on the X so that:

- The period problem is solved
- Certain mild compatibility conditions are satisfied

From Complex Analysis to Euclidean Polygons

The period problem is typically hard. Using flat structures, transfer the period problem to one involving Euclidean polygons and compute explicitly (algebraically!) the periods. To achieve this we:

- Assume (fix) some symmetries of the surface to reduce the number of parameters (and the number of conditions)
- Find a suitable class of polygons to study