On the Principles Which Turn Coffee into Theorems Although This Could Be a Very Long Title and That Would Also Be OK

by Ima Student, Bachelor of Science

A Thesis Submitted in Partial
Fulfillment of the Requirements
for the Degree of
Master of Science
in the field of Mathematics

Advisory Committee:

John Q. Faculty, Chair

Karl Gauss

Karl Gauss, Jr.

Graduate School
Southern Illinois University Edwardsville
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ABSTRACT

ON THE PRINCIPLES WHICH TURN COFFEE INTO THEOREMS ALTHOUGH THIS COULD BE A VERY LONG TITLE AND THAT WOULD ALSO BE OK

by

IMA STUDENT

Chairperson: Professor John Q. Faculty

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ACKNOWLEDGEMENTS

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CHAPTER 1

INTRODUCTION

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CHAPTER 2

PRELIMINARIES A VERY LONG TITLE WHICH SHOULD BREAK INTO TWO LINES IN THE TABLE OF CONTENTS

2.1 Riemann Surfaces and Differential Forms could potentially be a very long heading, which might cause problems, and we should check to see that this is properly indented

In this section, we define what a Riemann surface is and discuss differential forms on Riemann surfaces. To define a Riemann surface, we need a couple of definitions relating to complex manifolds.

2.1.1 This is a heading of another level

Here is some text. There might be mathematics here such as

$$\int_\Omega d\omega$$

or something else. In this section we'll illustrate some different headings and the like. Riemann surfaces are fun.

A lower heading And some more text.

2.2 Blah blah blah

asdfadsf

We now have the tools to define a Riemann surface.

Definition 2.1. A complex manifold X is called a Riemann surface if it is a one complex dimensional connected holomorphic manifold. [**?**]

In our last example, we showed that $\hat{\mathbb{C}}$ is a complex manifold. Since $\hat{\mathbb{C}}$ is connected and has one-complex dimension, $\hat{\mathbb{C}}$ is a Riemann surface. Riemann surfaces will be used as domains of the minimal surfaces discussed in this thesis. A special type of Riemann surface if formed by the solution set of a two variable polynomial equation. These types of Riemann surface will be useful later on. The following proposition describes these types of Riemann surfaces.

Proposition 2.2. *Fix eight unique points $x_i \in \mathbb{C}$. The set*

$$X = \left\{ (x, y) \in \hat{\mathbb{C}}^2 : y^2 = \prod_{i=1}^{8} (x - x_i) \right\}, \tag{2.1}$$

is a Riemann surface.

We must note that the point (∞, ∞) is a solution to the equation and is a point in X.

Proof. Put

$$P(x) := \prod_{i=1}^{8} (x - x_i).$$

First, we must show that X is a complex holomorphic manifold. Since X is a subset of a Hausdorff space $\hat{\mathbb{C}} \times \hat{\mathbb{C}}$, we have that X is Hausdorff (using the metric topology). We now need to build the complex manifold structure. For points (x, y_0) such that $y_0 \neq 0, \infty$ we take the open set $B_\epsilon(x) \times B_\epsilon(y_0)$. We want to find an ϵ small enough so that the projection $\pi_1(x, y_0) = x$ is a homeomorphism. Let ϵ_1 be chosen small enough so that $B_{\epsilon_1}(y_0)$ does not contain $-y$ for all $y \in B_{\epsilon_1}(y_0)$. Let ϵ_2 be chosen small enough so that $x_1, x_2, ..., x_8$ are not included in $B_{\epsilon_2}(x)$. Put $\epsilon = \min\{\epsilon_1, \epsilon_2\}$. Then π_1 is bijective in $B_\epsilon(x) \times B_\epsilon(y_0)$. Since π_1 is bicontinuous, π_1 is a homeomorphism. For points $(x, 0)$ we need different open sets. Since $P(x)$ has eight unique zeros. we can see that

$$P'(x) = \sum_{i=1}^{8} \prod_{j \neq i} (x - x_j)$$

is nonzero at these zeros of $P(x)$. The Implicit Function Theorem guarantees us small neighborhoods about these points such that the projection $\pi_2(x, y) = y$ is bijective. For the point (∞, ∞), we take the function $\pi_\infty(x, y) = \frac{1}{x}$, where $\pi_\infty(\infty, \infty) = 0$ as the local coordinate. The open set that we define the local coordinate in is

$$\{(x, y) : |x| > |x_i|, i = 1, ..., 8\}.$$

This projection is bijective. Now let U be be a set in the cover of the type containing a point (x, y) such that $y \neq 0$ and V be the type in the cover where it contains a point $(x, 0)$. If $U \cap V \neq \emptyset$, the mapping

$$\pi_2 \circ \pi_1^{-1}(x) = \sqrt{\prod_{i=1}^{8}(x - x_i)}$$

is single valued, bijective, and holomorphic. Thus, we have shown that X is a complex manifold. Now we show that X is a Riemann surface. To show that X is connected, we show that X is path connected. Let (x_1, y_1) and (x_2, y_2) be two points. We project to the y coordinate. Since $\sqrt{P(x)}$ is continuous, we can travel along this path until we arrive at y_2. When we inverse project, we will either be at (x_2, y_2) or $(-x_2, y_2)$ since the square root is a multivalued function. If the later occurs, we can make an analytic continuation. We do this by looping once around a zero. Then, when we inverse project, we arrive at (x_2, y_2). Hence, X is path connected. Now we show that X has one complex dimension. When developing the complex manifold structure, we showed that the projections π_1 or π_2 are homeomorphisms in small enough balls around any point. Thus X is locally homeomorphic to $\mathbb{C}$. Since X is connected and locally homeomorphic to a one-dimensional complex space, X has one complex dimension. Hence X is a Riemann surface. $\square$

CHAPTER 3

CONCLUSION

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REFERENCES

[GBW96] Karsten Große-Brauckmann and Meinhard Wohlgemuth. The gyroid is embedded and has constant mean curvature companions. *Calc. Var. Partial Differential Equations*, 4(6):499–523, 1996.

[Tra06] Cindy Traub. *Topological Effects Related to Minimum Weight Steiner Triangulations*. PhD thesis, Washington University, May 2006.

APPENDIX A

First One

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APPENDIX B

Second One

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