Deformations of the gyroid and Lidinoid minimal surfaces

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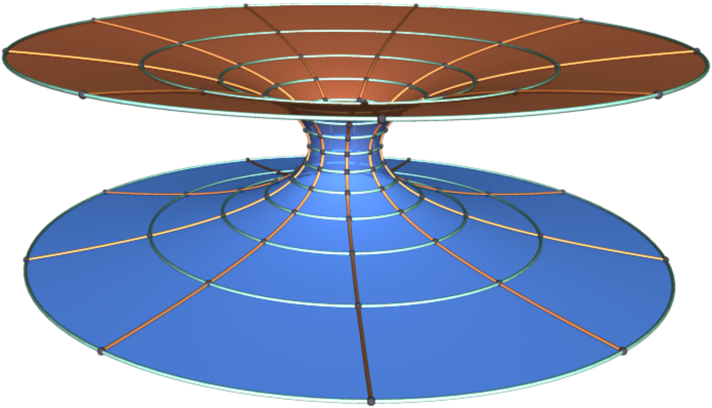
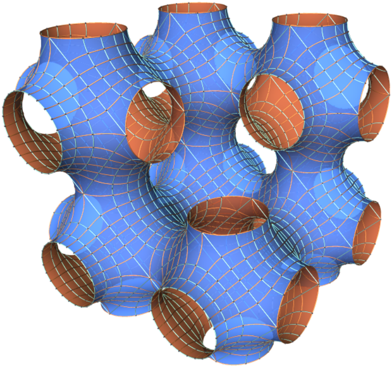
Definition of a minimal surface

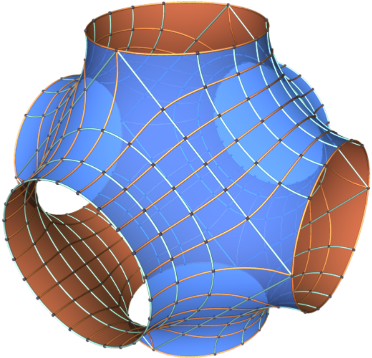
A minimal surface is a 2-dimensional surface in $\mathbb{R}^3$ with constant mean curvature $H \equiv 0$.



Definition of triply periodic minimal surface

A triply periodic minimal surface is a minimal surface M immersed in $\mathbb{R}^3$ that is invariant under the action of a rank 3 lattice Λ.





If M is embedded, the quotient $M/\Lambda \subset \mathbb{R}^3/\Lambda$ is compact.

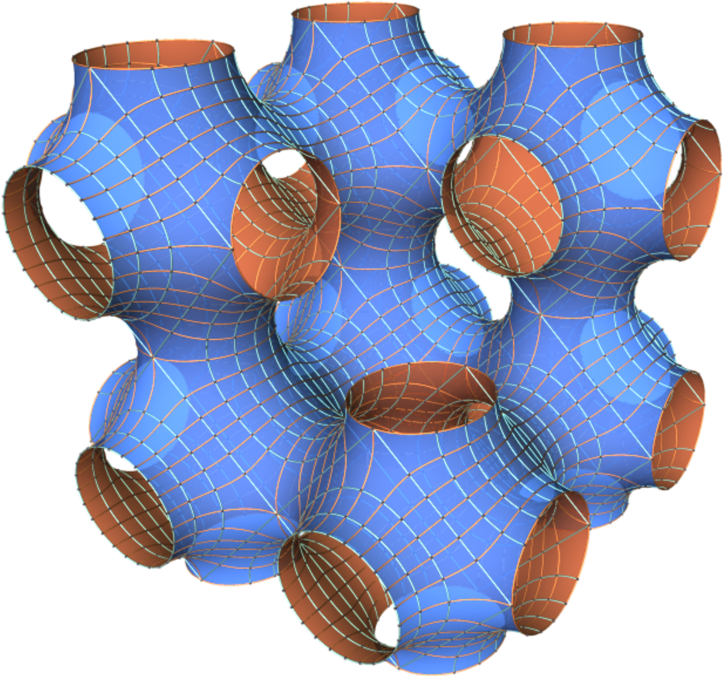
Large family of examples

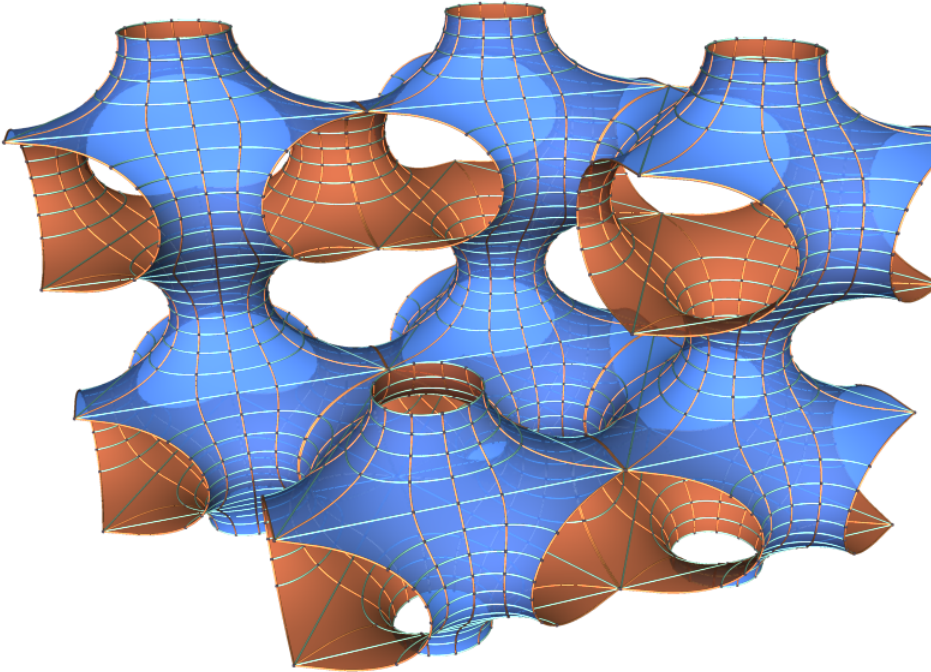
Since M/Λ is compact (for an embedded TPMS M), it has a well-defined (finite) genus.

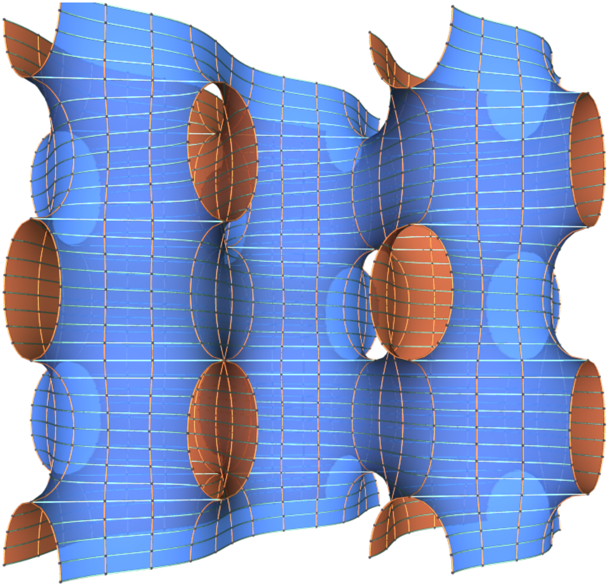
Theorem (Meeks '75)

Every TPMS in $\mathbb{R}^3$ has genus at least 3.

Many classical examples have genus 3.







Meeks' surface family

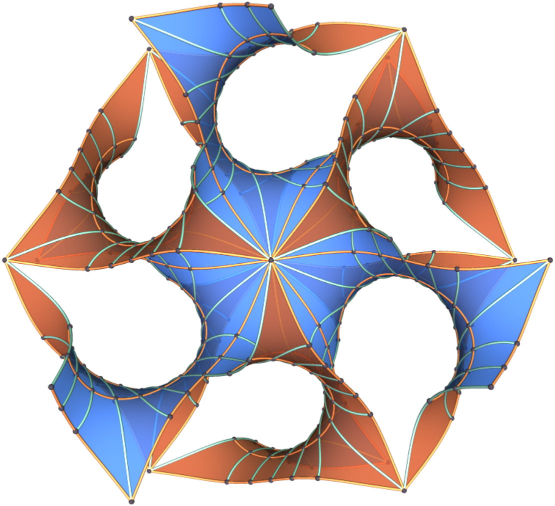
Theorem (Meeks '75)

There is a continuous 5-parameter family of embedded triply periodic minimal surfaces of genus 3 in $\mathbb{R}^3$.

In other words: any member of this 5-parameter family can be continuously deformed into any other member of the family through embedded minimal surfaces by changing the lattice.

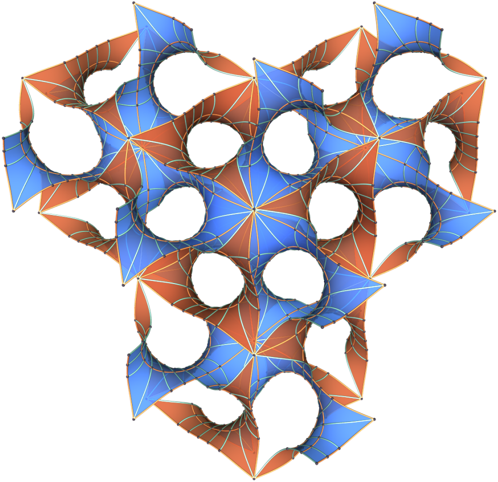
All known examples of triply periodic minimal surfaces of genus 3 are in the Meek's family — with only 2 exceptions: the gyroid and Lidinoid minimal surfaces.

The gyroid



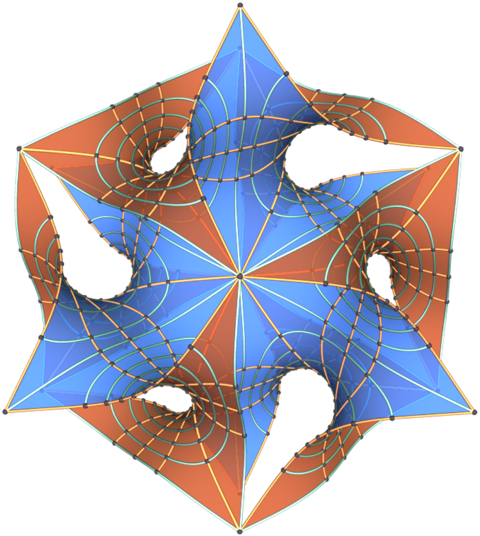
- Discovered in 1970 by Alan Schoen (a NASA scientist)
- Contains no straight lines or planar symmetries (first example)
- Is "associate to" the *P* surface

The gyroid



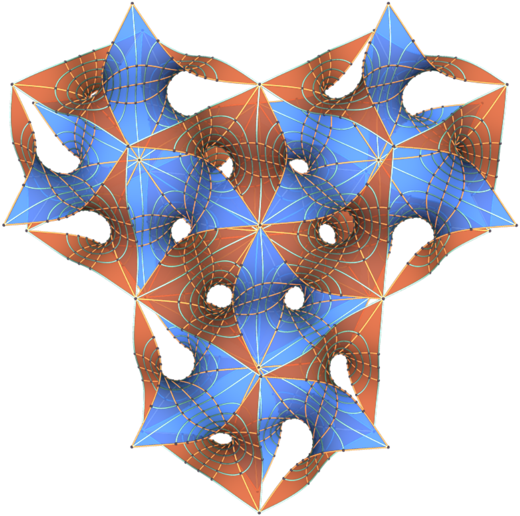
- Discovered in 1970 by Alan Schoen (a NASA scientist)
- Contains no straight lines or planar symmetries (first example)
- Is “associate to” the *P* surface

The Lidinoid



- Discovered in 1990 by Sven Lidin (a chemist / crystallographer)
- Like the gyroid, contains no straight lines or planar symmetries
- Is "associate to" the *H* surface

The Lidinoid



- Discovered in 1990 by Sven Lidin (a chemist / crystallographer)
- Like the gyroid, contains no straight lines or planar symmetries
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Main Theorem

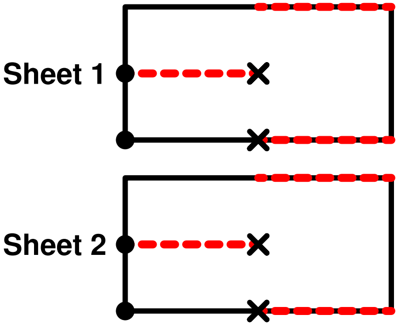
Theorem (W, 2006)

There is a one-dimensional continuous family of embedded triply periodic minimal surfaces of genus 3 that contains the gyroid and preserves an order 2 rotational symmetry (throughout the family).

There is an additional one-dimensional family that contains the gyroid and preserves an order 3 symmetry, and yet another one-dimensional family that contains the Lidinoid and preserves an order 3 symmetry.

~~Sketch~~ Flash of the technique

All genus 3 TPMS with a rotational symmetry can be parameterized using holomorphic maps from a flat, branched torus. The holomorphic map can be made to depend only on the conformal type of the torus and the “angle of association”.



To determine which tori / angles of association give embedded surfaces, need to solve the "period problem" (i.e., make sure that closed curves on parameter torus actually close in space). It is possible to explicitly understand the period problem using "flat structures" a technique which transfers this difficult analysis problem to a problem involving only algebra and simple Euclidean geometry.

Forthcoming work

Preliminary investigation suggests an additional Lidinoid family (which would make the results for the Lidinoid and gyroid) identical. This additional family likely has a significantly different conformal structure.

We conjecture that a 5-parameter family of gyroids exists (similar to the Meeks family).