A REAL-LIFE DATA PROJECT FOR AN INTERDISCIPLINARY MATH AND POLITICS COURSE

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Abstract: We describe a non-statistics, data-based activity developed by the author for an interdisciplinary course on mathematics and politics that uses actual ballot data from the City of San Francisco, California. The assignment is appropriate for a general education audience and develops students' ability to work with data, improves their understanding of functions and algorithms, and requires them to make (and support) choices grounded in logic and mathematics. We also provide a Mathematica notebook that makes it easy to take data from San Francisco to create such an assignment.

Keywords: social choice, real-life data, politics, liberal arts mathematics, voting

1 INTRODUCTION

The author's institution has a novel general education requirement; students must take one "interdisciplinary studies" course at the 300-level. These courses nearly always feature team-teaching between two faculty members from different departments. A colleague (Prof. Ken Moffett) from the Department of Political Science and the author (from the Department of Mathematics and Statistics) teach an interdisciplinary studies course on Mathematics and Politics. The wonderful text [6] serves as a general resource from we which teach some chapters; we supplement with other materials from our disciplines.

The course has no prerequisites other than that students hold junior-standing, but we can assume that students have completed most of the general education curriculum and therefore have basic facility in writing. Students may not have had mathematics in college but generally have had some sort of a reasoning or logic course (although this course typically would not have been taught by the math department). We have about 75 students in the course; they come from a large variety of majors (engineering, mathematics, sciences, humanities, education, etc.) Although in theory students get to choose an interdisciplinary course that interests them, in practice students usually take what is available and fits their schedule. Because of the type of students we receive, the assignment described below is applicable to similar courses such as quantitative literacy, liberal arts mathematics, and other courses stressing writing and reasoning.

One of the author's broad goals in the course is to show the students how a theoretical mathematician might think about and frame an issue. In doing so, I hope to debunk the students' initial notion that a mathematician sits around all day adding fractions. As part of this, I expect the students to develop the ability to think logically about the world and to be able to write and understand simple proofs.

One of the assignments that we have developed for the course uses actual real-world ballot data. We ask students to make choices about the data, analyze the data, and write a report. Their analysis is entirely non-statistical and requires them to understand the data, understand several decision making rules, and implement these rules using a spreadsheet program. Although we developed the assignment for a course specifically on math and politics, these topics are also treated in several quantitative literacy / liberal arts mathematics books such as [1] and [2].

In this article, we describe the assignment and describe a Mathematica program that we developed for the instructor to make the assignment easy to create. First we explain the basics of social choice procedures. Then we explain the assignment and some of reasons why we find the assignment so compelling. At the end of this note we provide links to the Mathematica notebook and data sets for you to implement a similar exercise in your classroom.

2 BRIEF OVERVIEW OF SOCIAL CHOICE THEORY

In the course, we cover several different social choice procedures ("voting methods"), among them: plurality, Borda count, instant runoff voting, and Condorcet's method. We describe these methods briefly, but see [6] for a more detailed description of these methods. Loosely, a social choice procedure is a method that determines a social choice (winner or winners of an election) based on a set of ballots. Formally, given a finite set of alternatives A, a ballot is a permutation of A which we interpret as a ranked preference list. A *social choice procedure* is a function that takes as input a finite set of ballots and produces as output a (possibly empty) subset of A. The *social choice* (or winner) of the election is the output of the social choice procedure (unless the set is empty, in which case we say there is no social choice).

2.1 Plurality

The plurality procedure produces the candidate with the most first place votes (this can be a set of candidates in the event of ties). Although most of the United States uses plurality voting, plurality voting has been criticized for one of its most serious weaknesses — the problem of vote splitting. To illustrate, consider the following set of ballots:

Voters 1-3	(a,b,c)
Voters 4-5	(b,c,a)
Voters 6-7	(c,b,a)

For these ballots, a is the plurality winner, even though 4 out of 7 voters prefer anything else to a. This phenomenon is more likely to occur when the number of alternatives is large and when some candidates are ideologically similar. We see vote splitting in modern American politics. Many political commentators believe that in the 2000 U.S. Presidential election, vote splitting may have played a role. Candidates George Bush and Al Gore were in a close race in the State of Florida, which, because of the electoral college system, was likely to decide the race. While there were several candidates from outside the two main political parties, the

Green Party candidate (Ralph Nader) received a significant number of votes (approximately 100,000, with the margin between Bush and Gore at around 600). Since the Green Party tends to be a liberal party, it is likely that these voters would have preferred Gore to Bush, however, their first place votes went to Nader. It is likely that the use of a different voting system in Florida would have resulted in the election of a different candidate in 2000.

2.2 Condorcet

The Condorcet method was proposed by Condorcet in his 1785 disquisition; at the time, his work was poorly regarded by mathematicians and received few positive reviews. The Condorcet method outputs the set of alternatives which pairwise beats or ties every other alternative. The Condorcet method is an attractive choice since a Condorcet winner is "clearly" the will of the people — no other candidate can beat the winner head to head. Unfortunately, if no candidate beats every other candidate head to head, then the Condorcet method does not output a social choice. Some methods (such as Copeland's method) [3] attempt to repair this deficiency by counting the number of head to head wins and outputting those alternatives with the most head to head wins. This system is familiar to sports fans as the way that many round robin tournaments are run.

2.3 Borda

Borda proposed this system in 1781; it was rediscovered by Laplace in 1795 and has appeared in many variants since then. The Borda count is a positional system; alternatives are given points based on their ranking on each ballot. If there are n alternatives and each voter is required to rank all alternatives, we give $n-1$ points for a first place vote, $n-2$ for a second, and so on. The alternative with the most points is declared the social choice. Saari [4] has argued that the Borda count is the "best" social choice procedure available for widespread use.

2.4 IRV/Hare/STV

The instant runoff voting (IRV) system, Hare system, and single transferable vote (STV) system appear in political systems around the world in different variations; we describe here one possible system. Suppose there are n alternatives; voters rank the alternatives and the alternative(s) with the fewest number of first place votes is eliminated from all ballots. Each ballot now expresses a ranking of the remaining $n-1$ alternatives. The procedure is repeated by deleting the alternative(s) with the fewest number of first place votes. We continue in this fashion until an additional round would eliminate all alternatives. The alternatives remaining are declared the social choice.

It is easy to construct examples with a small number of ballots so that the four procedures we have described give four distinct winners. Since these examples have only a dozen or so ballots the winners can be computed by hand. On the other hand, students are left with the feeling that the examples are contrived; can it *really* matter in real-life what the voting method is? We discuss in class the possibility that the winner of the 2000 election would have been different with a different social choice procedure, but since data about voter's preferences is not available our discussion is speculative.

3 REAL-WORLD DATA

The social choice procedures described above (with the exception of plurality) require *ranked preference lists*, not simply voters' top choice. If we seek to use real world data with our students we need access to ranked preference lists! In 2002, the City of San Francisco, California, switched from a several round runoff model to instant runoff voting for most local elections such as Mayor and member of the Board of Supervisors; the system was first used in 2004 [5]. As a result, San Franciscan voters no longer vote for their top choice alone in the voting booth; the voters rank their top three candidates. Most importantly for our purposes, San Francisco publishes data since 2008 online, including the raw ballots from the elections. (Although various localities use instant runoff voting, we are not aware of other localities that publish their entire data sets online.) Ballot data for previous elections is available in the Department of Elections' Election Archive, available at: `http://www.sfgov2.org/index.aspx?page=1671` For example, the first six lines of a ballot file are:

```
000000600001706700000040020000274001000014900
000000600001706700000040020000274002000015700
000000600001706700000040020000274003000015700
000000600001707400000130020000256001000016300
000000600001707400000130020000256002000015700
000000600001707400000130020000256003000014700
```

In this form the data isn't very useful, but a simple script (ours is written in Mathematica, but any scripting language will do) can convert the snippet above into:

```
d,l,l
r,l,b
```

Note that `b`,`d`,`l`, and `r` stand for actual candidates in the election; the first voter, for example, placed candidate "`d`" first and candidate "`l`" both second and third. The data, now in Comma Separated Value format, can be easily manipulated by any spreadsheet program.

Of particular interest to us — the data is *not contrived*; these are the actual ballots from an actual election. We know of no other source for such data. With some inspection, one can find San Francisco elections where the plurality winner is different from the winner under the other three methods mentioned above. For example, in the November 2010 election for District 10 Board of Supervisors, Lynette Sweet was the plurality winner, although Malia Cohen was the winner using Borda, Condorcet, and instant runoff (and therefore also the actual election winner). In fact, Cohen was third in the plurality rankings! A similar situation occurred in District 2 the same year.

It is a time consuming process to compute all of the winners using Excel for many different elections. We have created a Mathematica notebook that computes the winners using the methods mentioned here. The notebook also makes it possible to create data sets for students to analyze "what if" scenarios. For example, suppose that in the District 10 election mentioned above, several of the "minor" candidates band together to form one or more coalitions. There are examples of such coalitions that yield different plurality, Condorcet, and IRV winners! The same example yields different Borda winners depending on how the data is interpreted.

At `http://www.siue.edu/~aweyhau/mathandpolitics.html` we provide the Mathematica notebook, several CSV files of data, and an archive of several raw data files from San Francisco. New files are generated by San Francisco every election, so be sure to watch for new data every November!

4 STUDENT PROJECT

Here we describe the assignment and explain what goals of our course are achieved by this project. We also give examples of some typical student products and explain how we grade the assignment.

4.1 Assignment description

For the assignment, we ask students to take the data from San Francisco and compute the hypothetical election winners under plurality, instant runoff, Borda, and Condorcet's method using a spreadsheet program. The computation of plurality and Borda rely only on "counts" in each column or row and so the computation is fairly straightforward. Instant runoff must generally be done by a more time consuming method. Condorcet's method is not too difficult if one uses the logical functions AND and OR in Excel.

As one knows by watching almost any television program, one should not assume that voters are "rational", and so the ballot data can be messy. The following ballots are typical of those observed: (a,␣,b), (␣,a,a), (␣,␣,a), (␣,␣,␣), (a,b,a) where ␣ indicates a blank position in the preference list. It is not immediately obvious how one should interpret these ballots and how one should compute the various winners. San Francisco publishes a set of rules on how they deal with such ballots using the instant runoff system, but since they do not use Borda or Condorcet no such rules exist. Students, therefore, learn that real world data is messy! There are various ways to deal with these problems. For example, the ballot (␣,a,a) can be interpreted as (a,␣,␣), as (a,a,␣), or as (␣,a,␣). In fact, we have seen students use all of these interpretations. To enable the students to make these choices, we provide them four sets of data: the raw data, the raw data with blanks and duplicates removed, the raw data with blanks removed, and the raw data with duplicates removed. (These correspond respectively to the interpretations above.) Some students feel that different social choice procedures call for different data sets, while others believe a single data set makes the most sense. There is no right answer here, but students must argue for their choice and explain why their choice is reasonable. For example, a student could argue that using the Borda count, we should interpret the ballot as if it were (a,␣,␣), because it is 'clear' that the voter prefers a to any other choice. On the other hand, a student could reasonably argue that we should interpret the ballot as (␣,a,␣), because the voter intentionally did not rank a as highly as possible (the best student answers would discuss whether or not it would be rational to vote this ballot under the Borda system). It would be difficult for a student to provide a convincing argument that (␣,a,a) is a reasonable interpretation of the ballot; this effectively gives their voter more representation than if the voter had submitted a 'proper' ballot. Students must also realize, and we typically mention during the lecture, that if a voter votes (a,b,c), the voter clearly prefers a to d. Students must understand this 'unstated preference' in order to correctly compute the Condorcet winner.

Decision on how to deal with ballots are just one of several choices that the students must make. They also need to determine how they will award points using the Borda count. For example, if the ballot is (a,b,c), students need to determine how many points a should receive. We have seen students make a number of reasonable (and several unreasonable!) choices. Some excellent responses even compare the outcomes using several different ways of assigning points.

For the assignment, we ask students to submit a spreadsheet that shows their calculation of the winner of the election using each social choice procedure. They also write a paper that justifies the data set they used to compute the winners. Additionally, their paper addresses the following two questions:

- It is often said that when systems other than plurality voting are used, the number of candidates increases dramatically. Does the data from San Francisco support this conclusion? Why or why not?
- What was the rationale for and against switching to IRV? Can you find arguments either for or against that are, in your opinion, flawed? Why? (News resources may be useful here.)

4.2 Goals of the assignment

We believe that the assignment accomplishes many of our course goals, in particular, the assignment:

1. *requires students understand the social choice procedures and be able to implement them algorithmically.* Students who successfully complete this project must have a thorough understanding of the four social choice procedures used in this project. Moreover, they must develop a way to implement these procedures in a large scale, algorithmic way because of the size of the data set and the number of alternatives available.
2. *gets students comfortable using technology to work with very large data sets.* Most of the data sets from San Francisco district elections have at least 15,000 votes and so can not be tallied by hand. This project gives the students some familiarity using Excel on a large data.
3. *shows students that real-world data (and problems) are messier than those found in textbooks.* Textbook examples only discuss completely rational ballots such as (a,b,c). Students see that real-life

voters do irrational things like vote (a,b,a). Students realize that a number of difficult choices must be made in order to conform real-life data to the social choice procedures we discussed in class.

4. *demonstrates a key point of our course: that rules matter.* A major theme of our section on social choice procedures is that the choice of election method we use has signiicant affects on the outcome of an election. If chosen wisely, the data set will indicate that the winner of an election can change if the social choice procedure changes.
5. *requires that students make choices that are grounded in logic and then explain (argue) those choices.* The social choice procedures that we discuss in class have very specific rules, but those procedures have only been defined on complete preference lists with no duplicates or blanks. The real-life data is not so clean, and students must chose how to deal with them. They are graded partial on their ability to argue that their choices are reasonable.

4.3 Typical student product

Student papers are typically 6 to 10 pages long and usually have a few tables explaining the outcomes using the four different methods. As one might expect, the quality of these papers varies considerably. For example, consider the following student responses to the first question (about the number of candidates increasing). These are only a few sentences of what is sometimes a longer argument; the responses are reproduced as submitted but internal citations have been removed.

> I think that this is true in the San Francisco election. There were so many candidates in this election and I think that people were swayed by all the different messages and what other were voting. I also think it might not be true because of the way people voting.
>
> – Student 1

> The data from the November 2, 2010 election for San Franciscos Consolidated General Election had twenty candidates on the

ballot for the in the 10th district for Board of Supervisors. In previous years, the ballot had far less candidates. In November 2002, only 1 candidate was on the ballot for the District 10 Board of Supervisors. On the ballot in 2006, after IRV voting had become the social choice procedure, the number of candidates jumped up to 7 candidates. Finally, the Consolidated General Election in 2010 showed a jump to twenty candidates. With this data supporting the claim, it seems that when a method other than plurality voting is used to determine election winners, the number of candidates in an election increases.

– Student 2

The data from San Francisco does not always support the conclusion that when systems other than plurality, more candidates will run for election. In the November 2000 election for the 5th District Board of Supervisors, I found that that 11 candidates ran for election. In November 2004, 22 candidates ran, and in 2008 eight candidates ran. These elections show that more candidates ran for election after the voting system was switched. From table A, I found that two of the six districts candidates actually decreased from 2000 to 2004, which was during the vote system changing. I believe this shows that when changing the voting system to something other than plurality, candidates will increase but not dramatically.

– Student 3

Student 1's response does not meet our expectations. It provides no analysis and does not offer any data to support the student's conclusion. Student 2's response meets our expectations. The student uses external data to support the conclusion and writes appropriately. Student 3's response exceeds our expectation. This response is clearly thought out, uses data from various districts, organizes the data in a clear way, and draws a nuanced conclusion.

The second question above (about the rationale for and against switching to IRV) usually elicits longer responses and so it is difficult to reproduce them here. One particularly impressive response consisted of approximately four pages explaining various published arguments for and against switching to IRV in San Francisco. In a portion of the student's conclusion, she writes:

> A large claim that he makes is, "Rather than have the majority rule, Proposition A (switching to IRV) could actually reduce the actual number of voters who decide elections to a smaller portion than currently go to the polls in run-off elections." This is an interesting claim, but once again, this is all he says about it. He never goes on to explain what he means by this, nor does he offer any statistical evidence in support of the claim. His argument lacks the support needed to find it valid.

Would that all of our students analyze arguments tossed about in the media so critically!

Finally, we ask students their opinion of which social choice procedure is 'best' and ask them to justify their choice. Answering these questions requires that students think critically about how to make an argument and identify external sources to support their argument.

4.4 Description of grading

Finally, we make a few remarks about the grading of this assignment. My co-teacher and I typically both grade all of the papers; we usually divide the questions and attempt to identify the relevant parts in the papers that address the questions. We tell the students that their grade will be determined by:

1. The quality of your work.
2. The depth which with you explore your each question.
3. Whether you get the right answer for those questions for which such an answer exists.

4. Whether your spreadsheet clearly and sufficiently documents the procedures that you used to arrive at your answer. Your spreadsheet should allow us to easily determine the way by which you have arrived at your answers.

Students struggle the most with the question that asks them whether or not the number of candidates has increased after San Francisco's switch to IRV. Students feel instinctively that if there are 20 candidates there *must* have been an increase. Students who look at all of the data available see, however, that the connection is not so clear. For the most part, students are able to successfully compute the correct hypothetical winners using the four social choice procedures, and their spreadsheets are generally fairly easy to understand.

As the mathematician, I typically grade the more quantitative parts of the assignment. We tend to grade the assignment holistically. If an assignment meets our expectations and does not suffer from too many grammatical mistakes, it typically receives full credit. My co-teacher and I discuss ahead of time standard deductions for arguments that are not fully explained, and we typically make extensive comments on student work. Students have an opportunity to submit changes in response to our comments to partially improve their score.

5 CONCLUSION AND STUDENT FEEDBACK

Many students find the assignment challenging. Based on end-of-term comments on student evaluations, we believe that there are three common student concerns, represented by the verbatim comments:

> Not pleased with the workload. A little too much required for such a course.
>
> Excel work sheets were hard and little was given for them.
>
> Direction was sometimes lacking with respect to homework assignments.

These comments are representative of student complaints about the assignment. The first complaint is, we believe, caused by an initial (and mistaken) impression among students that these interdisciplinary courses are blow-off courses that are irrelevant to their major. We are quite upfront with students that we expect work from them commensurate with a junior level course. We do not plan to make changes to the assignment based on this concern. The second concern is a common one from students, especially for those students without much quantitative background. Instead of spending time in class covering the basics of Excel, we plan to develop a series of short tutorials explaining the basics of sorting, counting, deleting, moving, and using the logical AND and OR functions. Finally, students sometimes complain that the assignment is not detailed enough. We believe that this may be caused by open-ended questions. In the past we have not done a good job of articulating the goals of the assignment, in particular, we have not explicitly told students that we want them to make choices that are grounded in logic and then argue their choices. In the future we plan to explicitly state these assignment goals so that students understand that the question is not intended to have a single correct answer. In particular, the Condorcet method of the assignment seemed to generate a lot of emails from students asking for help.

Although the students find the assignment challenging, they have indicated to the instructors that they like working with real-world data. One student evaluation comment said

> This class was surprisingly interesting ... seeing math in places other than proofs and tests and in the real world, applications was really nice.

We are also pleased that the assignment lead to a number of discussions with students in office hours and after class about the real-life affects of social choice theory.

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APPENDIX: MATHEMATICA PROGRAM TO COMPUTE SOCIAL CHOICES

```
(* Comments appear inside paren/asterisk pairs *)

Clear[Evaluate[Context[] <> "*"]]

(* Data input and setup *)

votes = ReadList["voting/D10/BallotImage-D10.txt"];

(* turns the data into a list for ease of use with Mathematica *)
listVotes = IntegerDigits[votes];
```

```
(* Pull out only the candidate ranks for each ballot. *)

numPrefList =
  Table[{listVotes[[i]][[36 ;; 37]] , listVotes[[i + 1]][[36 ;; 37]] ,
    listVotes[[i + 2]][[36 ;; 37]] }, {i, 1, Length[listVotes], 3}];

(* Next command takes the candidate code and turns it into a,b,c, etc.
You can do this automatically or manually with the code that is
commented out.  If you want to collapse several candidates into one,
this is the place to do it. *)

alphaPrefList = numPrefList/.Thread[Rule[Union[Flatten[numPrefList,1]],
Take[Prepend[Map[Symbol,CharacterRange["a","z"]],""],
Length[Union[Flatten[numPrefList,1]]]]]];

(* alphaPrefList =
numPrefList/.{{0,0}->"",{2,6}->a,{4,6}->a,{4,7}->c,{4,8}->d,{4,9}->e,
{5,0}->f,{5,1}->g,{5,2}->h,{5,3}->a,{5,4}->a,{5,5}->a,{5,6}->l,
{5,7}->m,{5,8}->a,{5,9}->o,{6,0}->p,{6,1}->a,{6,2}->r,{6,3}->s,
{6,4}->a,{6,5}->u,{6,6}->a};  *)

(* Functions to clean the data. *)

(* creates a list of those alternatives which appear on ballots *)

alternatives = DeleteCases[Union[Flatten[alphaPrefList]], ""];

(* an internal function used to ensure that all alternatives are
represented in tallies *)
standardizeList[list_, alts_] := Module[{output = list, i},
   For[i = 1, i <= Length[alts], i++,
    If[! MemberQ[output, {alts[[i]], _}],
     PrependTo[output, {alts[[i]], 0}]]];
   Sort[DeleteCases[output, {"", _}]]];
```

```
(* removes duplicates from a list of preference lists *)

noDups[list_] :=
  Map[Fold[If[MemberQ[#1, #2], Append[#1, ""],
  Append[#1, #2]] &, {}, #] &, list];
(* removes blanks from a list of preference lists *)

noBlanks[list_] := DeleteCases[Delete[list, Position[list, ""]], {}];
(* removes blanks and duplicates from a list of preference lists *)

noBlanksnoDups[list_] :=
  DeleteCases[Map[DeleteDuplicates[#] &, noBlanks[list]], {}];

(* Functions to compute social choices. *)

(* computes the Borda tallies *)

borda[list_] :=
  Sort[Map[{#[[1]]/6, #[[2]]} &,
    3 standardizeList[Tally[Map[#[[1]] &, Map[PadRight[#, 3, ""] &, list]]],
       DeleteCases[Union[Flatten[list]], ""]] +
     2 standardizeList[Tally[Map[#[[2]] &, Map[PadRight[#, 3, ""] &, list]]],
       DeleteCases[Union[Flatten[list]], ""]] +
     1*standardizeList[Tally[Map[#[[3]] &, Map[PadRight[#, 3, ""] &, list]]],
       DeleteCases[Union[Flatten[list]], ""]]], #1[[2]] > #2[[2]] &];

(* computes plurality tallies *)

plurality[list_] :=
  Sort[standardizeList[Tally[Map[First, list]],
    DeleteCases[Union[Flatten[list]], ""]], #1[[2]] > #2[[2]] &];

(* internal function to remove an alternatives from all ballots *)
```

```
remove[list_, elt_] := Map[(Delete[#, Position[#, elt]]) &, list];

(* computes the instant runoff winner *)

instantRunOff[list_] := Module[{win = plurality[list], len},
   len = Length[win];
   If[len == 1, win[[1]][[1]],
    instantOff[noBlanks[remove[list, win[[-1]][[1]]]]]
    ]];

(* given a list of preference lists and two alternatives a and b,
determines which is the winner of a head to head competition *)
vs[list_, a_, b_] := Module[{
   count = {0, 0},
   winner
   },

  Map[(count +=
      Which[Length[Position[#, a, 1, 1]] == 0 &&
        Length[Position[#, b, 1, 1]] == 0, {0, 0},
       Length[Position[#, a, 1, 1]] == 1 &&
        Length[Position[#, b, 1, 1]] == 0, {1, 0},
       Length[Position[#, a, 1, 1]] == 0 &&
        Length[Position[#, b, 1, 1]] == 1, {0, 1},
       Length[Position[#, a, 1, 1]] == 1 &&
        Length[Position[#, b, 1, 1]] == 1 &&
        Flatten[Position[#, a, 1, 1]][[1]] <
         Flatten[Position[#, b, 1, 1]][[1]], {1, 0},
       Length[Position[#, a, 1, 1]] == 1 &&
        Length[Position[#, b, 1, 1]] == 1 &&
        Flatten[Position[#, a, 1, 1]][[1]] >
         Flatten[Position[#, b, 1, 1]][[1]], {0, 1}]) &, list];
  winner = Which[count[[1]] < count[[2]], b, count[[1]] > count[[2]], a];
```

```
  winner
  ]

(* determines the condorcet winner, or outputs the
phrase "no winner" if none exists *)
condorcet[list_] := Module[{
   winner,
   condmatrix,
   alts = DeleteCases[Union[Flatten[list]], ""],
   condwinner = "no winner",
   i = 1,
   j
   },
  condmatrix =
   Table[If[i == j, 1, 2], {i, 1, Length[alts]}, {j, 1, Length[alts]}];
  While[i <= Length[alts],
   If[Times @@ condmatrix[[i]] > 1,
    j = Flatten[Position[condmatrix[[i]], 2]][[1]];
    If[SymbolName[vs[list, alts[[j]], alts[[i]]]] ==  SymbolName[alts[[i]]],
     condmatrix[[i]][[j]] = 1;
     condmatrix[[j]][[i]] = 0;,
     condmatrix[[i]][[j]] = 0;
     condmatrix[[j]][[i]] = 1;]
    ];
   If[Times @@ condmatrix[[i]] == 1, condwinner = i; i = Length[alts] + 1,
    If[Times @@ condmatrix[[i]] == 0,  i ++ ]];
   ];
  If[condwinner == "no winner", condwinner, alts[[condwinner]]]
  ]
```

BIOGRAPHICAL SKETCHES

Adam Weyhaupt received a B.A. in Mathematics and a B.S. in Mathematics and Computer Science from Eastern Illinois University in 2001 and the M.A. and Ph.D. in Mathematics from Indiana University in 2002 and 2006 respectively. He is interested in minimal surfaces and pretty much any problem that he can involve undergraduates with, including geodesics on polyhedra and problems at the intersection of math and politics. In his (rumored) spare time, he enjoys being outdoors with his spouse and two boys.

THANKS

We are grateful to the anonymous referees for numerous constructive comments which greatly improved the quality of this manuscript.