WHAT DO BUTTERFLIES, KETCHUP, MICROCELLULAR STRUCTURES, AND PLASTICS HAVE IN COMMON?

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What do butterflies, ketchup, microcellular structures, and plastics have in common? **The gyroid.**

Scientists and mathematicians have long studied soap films and bubbles. The mathematical model of a soap film is a minimal surface (the model is just that, a model, because it is simplified by ignoring the effects of gravity and some other physical phenomena). Soap film takes the shape it does, in part, because the surface tension and the pressure of air on both sides of the film cause the film to want to take the minimum amount of surface area necessary to span a region of space. Minimal surfaces are characterized by this property. Take a small sphere, and intersect it with a minimal surface. The boundary where the sphere and surface touch will be a curve. A minimal surface must have the property that the surface inside the sphere has surface area at least as small as all other surfaces which contain this curve. (This need not be true for all sizes of spheres but will be true for some, maybe very small, sphere.)

The study of minimal surfaces began in at least the mid-1700s, but the "golden age" of minimal surfaces was in the mid to late 1800s, when Plateau (who was blind from staring directly at the sun for 25 seconds for an experiment), Schwarz, and others studied minimal surfaces and made great advances. After a long dormant period, Alan Schoen (a U.S. National Aeronautics and Space Administration scientist studying super-strong, super-light structures) discovered the *gyroid* in 1970.

To start to understand the geometry of the gyroid, let's consider a more familiar example: the honeycomb lattice in the plane. This arrangement of hexagons is doubly periodic; there are two vectors v and w that generate the lattice. In other words, if one starts with a single hexagon and translates copies in the directions of v and w, we can generate as large a piece of the lattice as we wish. Another way of saying this is that the lattice is invariant under translations of v and w; if we translate a copy of the entire lattice by v or w, the copy will be perfectly superimposed on the original lattice.

The gyroid has a similar structure. It is *triply periodic*, meaning that a small piece of the surface may be used to assemble the entire surface by taking a fundamental piece and translating copies in three independent directions in space. Many examples of triply periodic minimal surfaces were known prior to Schoen's work, but all examples had been

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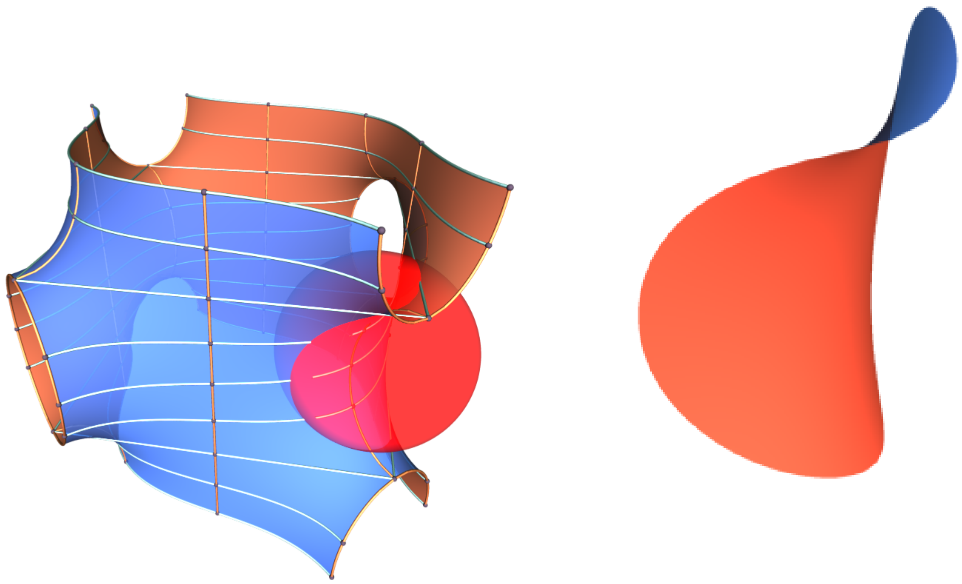

FIGURE 0.1. A small sphere intersected with a minimal surface, and the piece of the surface lying inside the sphere. The small piece has the smallest area among all surfaces with the same boundary.

constructed by finding the equation of a minimal surface that spanned a particular, nice boundary curve. (Think of the film formed by dipping a wire frame in soap solution.) To make the surface repeat in space, these wire frames were constructed of straight lines, with "nice" angles, and the surface was then formed by rotating about these lines and reflecting in planes. For example, the CLP surface has a wire frame shown below. However, the gyroid has no planes of reflectional symmetry and has no straight line segments lying on its surface. This makes the surface much more difficult to visualize.

But what really makes the gyroid interesting is that it seems to appear in all sorts of natural places! The force of air on two sides of a membrane results in a kind of competition; if there were no air "beneath" the surface then the air on "top" would push the surface down and vice versa, so a minimal surface is an equilibrium position. This same competition occurs other places in nature, and that may be why minimal surfaces are frequently observed by scientists.

Scientists recently announced in the Proceeding of the National Academy of Sciences that the beautiful iridescent colors in butterfly wings are caused by gyroids made of chitin and air. Another team of scientists used transmission electron microscopy to study the

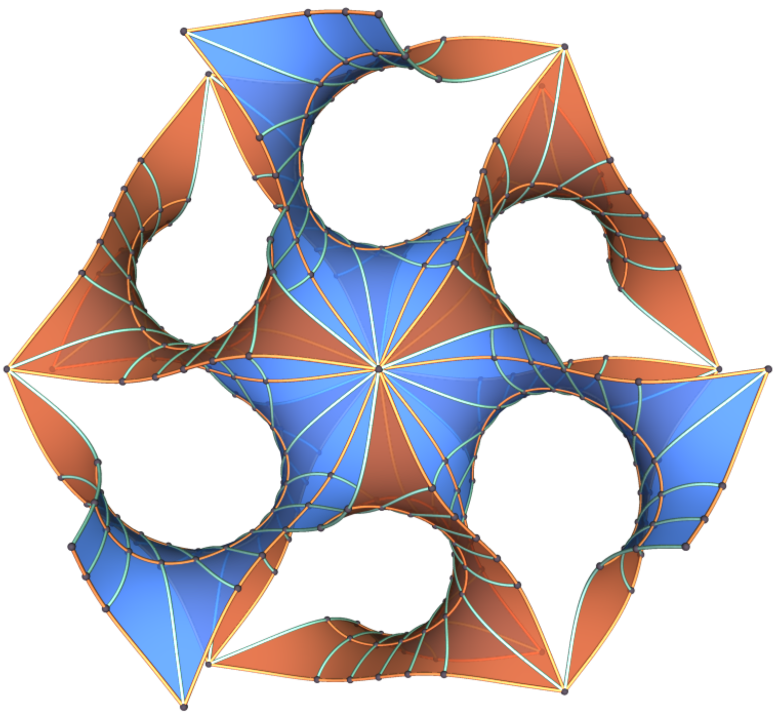

FIGURE 0.2. A translational fundamental domain of the gyroid surface

membranes inside mitochondria; they too found surfaces that appear to be the gyroid. A third team of scientists is using a giant, distributed supercomputer to study how highly viscous liquids like ketchup flow. In these liquids, the competition of amphiphilic (both water-attracting and fat-attracting) molecules results in a certain kind of molecular tension or competition; there too scientists are find spaces in these compounds made of gyroids.

A final interesting application is in "diblock copolymers". In a diblock copolymer, two large molecular blocks, say A and B, appear. In some cases, these blocks may be bonded together in the copolymer into long chains, but they may also repel themselves or the other molecule. This again causes a kind of competition, and scientists have long observed gyroids in the spaces between these molecules. Molecules may even have the ability to self-assemble into a gyroid, which could have future applications in nanostructures.

You can play with a rotatable gyroid and view movies at [1].

Mathematicians and scientists who study minimal surfaces have many tools at their disposal. Before computers were ubiquitous, one would often build a physical model out of

[1] I am happy to make these resources available through my website or any other convenient venue.

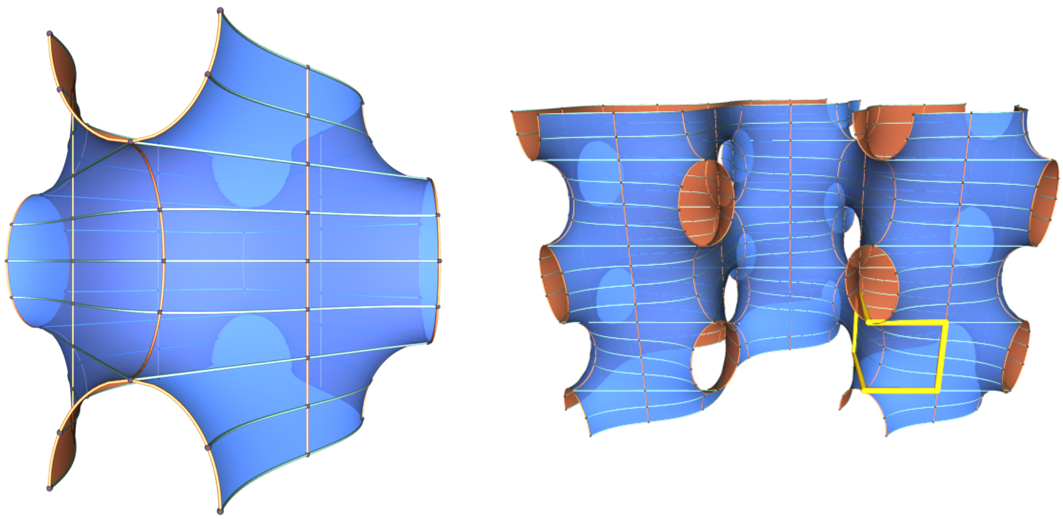

FIGURE 0.3. A fundamental domain of the P surface, with a wire-frame highlighted in yellow. On the right, several copies of the fundamental domain.

soap, plastic, or paper to help visualize the surface. Now, of course, we often use computers to develop rotatable models. While these are good for our intuition, to prove our results we must use some of the many mathematical tools available from the fields of:

a) Differential geometry
b) Complex analysis
c) Partial differential equations

Like much of modern mathematics, the best results come from a combination of these approaches.

Plus has written before about soap films and minimal surfaces:

- http://plus.maths.org/content/kelvins-bubble-burst-again
- http://plus.maths.org/content/double-bubble-no-trouble
- http://plus.maths.org/content/swimming-mathematics
- http://plus.maths.org/content/getting-handle-soap

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