Asymmetric Correlations of Futures Markets and Optimal Hedging

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Abstract

This paper extends the analysis of correlation asymmetry to futures markets. Consistent with the findings on international equity indices and on US stock portfolios, we find that correlations between spot and futures returns show similar asymmetric pattern. We use extreme value theory to model the dependence between spot and futures returns in ten markets including stock indexes, commodities, and foreign exchanges. Regarding stock index related contracts as well as crude oil futures, we find that correlations between spot and futures returns are much greater on the downside, especially for extreme moves, than on the upside. We then study the implications of correlation asymmetry on optimal hedge ratios and the resulting hedge performances for short and long hedgers. Our analysis indicates that hedgers who hold long positions benefit more from futures trading than investors who hold short positions in reducing downside risk. We also find that hedge ratios that aim to minimize downside risk yield generally better results than the traditional minimum variance hedge for the post-estimation sample.

Keywords: Risk Management, Correlation Asymmetry, Extreme Value Theory, Hedging, Downside Risk

***JEL* classification**: C12, C13, G12

1 Introduction

Understanding the relation between volatility and correlation is imperative for financial analysts who measure and manage market risks. In portfolio diversification, portfolio managers depend on the reliability of their estimates of how returns on assets in their portfolios are correlated. Similarly, in the context of hedging, risk managers interested in hedging exposure to risk factors depend on their estimates of correlations and volatilities for assets they use in their hedge portfolios. Correlations are also crucial for pricing and hedging derivative securities with payoffs that depend on more than one asset. Recent research on domestic and international stock markets suggests the notion of correlation asymmetry, i.e. correlations computed separately for ordinary and stressful market conditions differ considerably, a finding which suggests the existence of different regimes for stock return dependencies. In particular, correlations tend to be much greater on the downside, especially for extreme moves, than on the upside.

Correlation asymmetry has implications for several applications including portfolio diversification, risk management, and policy making. In the case of portfolio diversification, failing to take into account the increase in downside correlations leads to suboptimal portfolio weights, eroding the benefits from diversification in bear markets when it is most needed. If all stocks tend to fall together when the market falls, the value of diversification may be overstated by the portfolio manager who fails to take this into consideration, leading him to over-invest in risky assets. In the case of risk management, risk managers face the possibility that their hedges will be useless when they are most needed, namely during stressful market conditions. Furthermore, asymmetric correlations make it harder for risk managers to decide on what historical data should be used to estimate parameters for worst case analyses and to stress test their models. Finally, policy makers need to take into account the relation between volatility and covariances as their actions have the potential to cause volatility in financial markets which in turn may lead to unanticipated risks for investors.

A number of papers have tested the existence of correlation asymmetry in equity portfolios [see for example Bookstaber (1997), Loretan and English (2000), Bekaert and Wu (2000), Ang and Bekaert (2000), Ang and Chen (2002)]. The general finding is that correlations between stocks and the aggregate market are much greater on the downside than on the upside. Furthermore, the degree of co-movement gets even stronger during extreme market states. Ang and Chen (2002) analyze daily US equity returns between July 1963 and December 1998 and find that correlations between domestic equity portfolios and the aggregate market are greater in downside markets than in upside markets. Their analysis also indicates greater asymmetries among smaller stocks, value stocks, and recent losers. Longin and Solnik (2001) extend the analysis to international markets, however using a different methodology. Their analysis of monthly equity index returns for five countries over the period 1960-1990 leads to the finding that correlation is not related to market volatility but to market trend, i.e., conditional correlations increase in bear markets only. A similar finding is reported in Silvapulle and Granger (2001) who analyze 30 Dow Jones Industrial stocks between 1991 and 1999. Other related studies focus on cross-sectional dispersions of individual stocks that are documented to have increased over the past years [Campbell, Lettau, Malkiel and Xu (2001), Bekaert and Harvey (2000) and Duffee (1995)]. Separately, Chow, Jacquier, Kritzman and Lowry (1999) investigate the implications of asymmetric correlations on the selection of optimal portfolios. They present a procedure to identify multivariate outliers and use the outliers to estimate a new covariance matrix that they use to calculate the optimal portfolio weights. However, the notion of correlation asymmetry and its implications on risk management has not been fully extended to derivatives markets. Recently, Brooks, Henry and Persand (2002) analyze the impact of asymmetry on time varying hedges involving daily FTSE 100 stock index and stock index futures contracts over the period 1985 to 1999. They conclude that a model that allows for time variation and asymmetry in the variance-covariance matrix gives superior in-sample hedging performance. However, they also find that the simpler symmetric model is not outperformed in a hold-out sample.

In this paper, we study the conditional correlation structure of spot and futures returns across ten markets and analyze the implications of asymmetric correlations on optimal hedge positions as well as performance of these hedges. Clearly, this is an important issue for risk managers as unstable correlations make it difficult, perhaps impossible, to hedge exposure to a given risk factor by taking an offsetting position in another asset. So, from the risk manager's point of view, knowledge of the particular pattern of correlations under different market conditions is a major concern, since the effectiveness of hedging operations based on estimated correlations depend on how accurate those estimates are and how representative the estimation period is for the period when hedges are most needed. In a recent study, Guay and Kothari (2001) analyze a sample of 234 large non-financial corporations that use derivatives. They find that if the median firm simultaneously experiences a three standard deviation change in interest rates, currency exchange rates, and commodity prices, it will collect \$15 million of cash from its entire derivatives portfolio and that the entire derivatives portfolio will rise in value by \$31 million. The findings also suggest that these dollar amounts are modest relative to firm size, operating cash flows, investing cash flows and other firm benchmarks raising questions about the role of derivatives securities held by non-financial firms. However, one has to note that the use of derivative securities as hedging instruments does not mean that these securities are used optimally indicating the benefits of these markets will be fully realized. Similar to the argument made for portfolio optimization, the failure to take correlation asymmetry into consideration leads to suboptimal hedge positions for hedgers. The hedger will tend to understate the value of hedging leading him to take suboptimal hedge positions. In return, the hedger will benefit less from the futures markets in hedging his exposure to the risk factor. The analysis of correlation asymmetry is even more important for risk managers who are concerned with downside risk, i.e., the consequences associated with falling below a pre-specified target rate of return. For a hedger who aims to avoid falling beyond a target rate of return, failing to take higher downside correlations into account will lead to lower than optimal hedge positions and thus lower the effectiveness of these hedges. In a related paper, Ang, Chen and Xing (2001) define downside risk for a stock in terms of how its returns are correlated with the market conditional on downside moves of the market. Their analysis indicates that stocks with greater downside risk, measured by higher correlations conditional on downside moves of the market, yield higher returns even after controlling for the market beta, the size effect and the book-to-market effect.

Clearly, the effectiveness of the futures hedge depends on how well the returns on the futures contract track the returns on the spot. So, a risk manager's choice of the optimal hedge position depends on the reliability of his model of how spot and futures returns are correlated. In this paper, we use extreme value theory to identify extreme correlations between spot and futures returns across ten markets. We identify an extreme market not only by extreme high or low returns but also by unusual correlations among spot and futures returns. Longin and Solnik (2001) use extreme value theory to identify the correlation of extreme returns of monthly equity indices for five countries. However, they do not extend the analysis to how extreme correlations translate into optimal portfolio decisions. That is, having described how asset returns are correlated at different states of the market, we need to know how worse off one will be without taking this into consideration, i.e. how it would impact the benefits from global diversification.[1] A first contribution of this paper is to extend the analysis of correlation asymmetry to derivatives markets, in this case futures markets. The data set we analyze contains a number of markets including stock indices, foreign currencies, and commodities. Research on asymmetric correlations has focused mainly on asset returns from equity markets as well as foreign currencies. Therefore, it will be particularly interesting to see if this phenomenon applies to returns from commodity markets as these markets are fundamentally different from equities and foreign currencies. Second, we study the implications of correlation asymmetry on optimal hedge positions as well as the effectiveness of these hedges, an issue critical to risk managers. We argue that a risk manager who fails to take into account higher downside correlations between the returns on the spot and futures contracts will understate the value of these markets as hedging instruments. Third, we distinguish between short and long futures positions and use a general formula for downside risk to study how correlation asymmetry impacts the effectiveness of futures markets in reducing downside risk for these two types of positions. The reason we analyze short and long positions separately is that these investors will be interested in the opposite tails of the return distribution for their hedge portfolios. Our findings indicate that correlation asymmetry exists in futures contracts on stock exchange related indexes and crude oil. However, we do not find evidence on asymmetry in most of the commodities as well as foreign exchange contracts. Regarding the implications on optimal hedge positions and performance of hedges, we find that hedgers who hold long positions benefit more from futures trading than investors who hold short positions. Finally, we compare the post-estimation sample hedging performance of minimum variance hedges with hedge ratios that minimize the downside risk. Our analysis indicates that optimal hedge ratios that minimize the downside risk yield generally better results than the traditional minimum variance hedge. Although variability increases for minimum Lower Partial Moment (LPM) hedge portfolio returns, it is possible to choose an optimal target return for the LPM function so that a higher reward-to-risk ratio is achieved. In the case of S&P 500 futures, we find that choosing a target rate of one standard deviation below

[1] Ang and Chen (2002) briefly touch on the economic significance of asymmetric correlations under certain assumptions on utility.

the average spot return yields the best post-estimation sample performance for the hedge portfolio.

The remainder of the paper is organized as follows. Section 2 summarizes hedging with futures contracts and describes the two measures of risk that we use; variance and downside risk. It also presents some analytical results regarding the implications of conditional correlations on the choice of optimal hedge positions. Section 3 provides a brief explanation of extreme value theory for univariate and multivariate return distributions. It then outlines the procedure to estimate extreme correlations of futures and spot returns. Section 4 describes the data and presents empirical results on optimal hedge positions and hedge performances for short and long hedgers. Concluding remarks are given in Section 5.

2 Risk management using futures contracts

A futures contract is essentially a promise (and obligation) to buy or sell a specific amount of an asset at a certain time in the future for a certain price. Although futures markets were originally set up to meet the needs of hedgers, these markets have attracted many different types of traders. For hedgers, futures markets offer the opportunity to avoid the exposure to adverse movements in asset prices. For speculators, futures markets offer the opportunity to capitalize on their bets about prices in the future. These investors keep up the cash flow into markets hoping that they can buy contracts low and sell high. For arbitrageurs, futures markets offer riskless profit opportunities through simultaneous transactions in two or more markets. In this paper, we are interested in the use of futures contracts as hedging instruments.

Traditionally, variance of the hedge portfolio has been adopted as the appropriate risk measure that needs to be minimized. However, hedges that aim to minimize variance avoid both high and low returns which may not necessarily be consistent with the risk manager's goals. Recent evidence has suggested the notion of one-sided risk. For example, in a recent survey Adams and Montesi (1995) found that corporate managers are mostly concerned about downside risk. Earlier, Petty and Scott (1981) suggested that many Fortune 500 firms identify risk as the probability of falling below a target return. Correlation asymmetry is also an important issue for risk managers who are overly concerned with downside risk particularly when the market is bearish or unusually volatile. The remainder of this section describes the two measures of risk that we use in this paper.

2.1 The minimum variance hedge

Consider a short hedger who is endowed with an initial wealth W_0 and a given non-tradable spot position Q at time 0. Suppose that the hedger sells kQ futures contracts at time 0. At time 1, his wealth is $W_1 = W_0 + (\Delta p)Q - (\Delta f)kQ$, where Δp and Δf are, respectively, changes in spot and futures prices from time 0 to time 1. Alternatively, let p_0 and f_0 denote the spot and futures prices, respectively, at time 0. Then $r_p = \Delta p/p_0$ is the spot return and

$r_f = \Delta f / f_0$ is the futures return. The end-of-period wealth can be rewritten as

$$W_1 = W_0 + (r_p - hr_f)Qp_0$$

where $h = kf_0/p_0$ is the (adjusted) hedge ratio. To isolate the effects of W_0 and Q, we consider a general formula $r_{HP} = r_p + hr_f$ as the return on the hedge portfolio (HP). The hedger then chooses an optimal hedge ratio which minimizes the variance of the return on the hedge portfolio by solving

$$\min_h Var(r_{HP}|\Phi) = Var(r_p|\Phi) + h^2 Var(r_f|\Phi) + 2hCov(r_p, r_f|\Phi)$$

where Φ denotes the information set at $t = 0$. The optimal or minimum-variance hedge ratio, h^*, is then obtained by setting the derivative of the variance term with respect to h equal to zero and solving for h,

$$h^* = -\frac{Cov(r_p, r_f|\Phi)}{Var(r_f|\Phi)} = -\frac{\sigma_p}{\sigma_f}\rho_{pf} \tag{1}$$

where σ_p and σ_f are the standard deviation of spot and futures returns respectively and ρ_{pf} is the correlation between spot and futures returns, all conditioned on the information set Φ.

2.2 Conditional correlations and optimal hedging

The optimal hedge position as formulated in (1) is a direct function of the volatility and correlation estimates for the returns on the futures and spot contracts. Therefore, a good understanding of the relation between volatility and correlation is a major issue for the risk manager. Now we briefly demonstrate how correlations under different conditioning scenarios can affect the calculation of optimal hedge positions. Boyer, Gibson and Loretan (1999) provide a detailed analysis on how correlations might be linked to volatility. However, they do not deal with the implications on portfolio diversification. Consider a pair of bivariate normal random variables x and y with variances σ_x^2 and σ_y^2, respectively, and covariance σ_{xy}. Denote the unconditional correlation between x and y as $\rho_{xy} = \sigma_{xy}/(\sigma_x\sigma_y)$. Consider any event $x \in E$ where $E \subset \Re$ ($\Re$ the set of real numbers) such that $0 < \Pr(E) < 1$. $x \in E$ can be any conditioning rule which potentially affects variability of x, y or both (e.g. bear, bull, or an extreme market condition however one defines it). Let, for example, $x \in E$ denote cases where x is in the 1% lower tail (this can be used to define a bear market condition if x is used to denote the return on the market index). For simplicity, we will use E to denote any event of the type $x \in E$. Then, the conditional correlation $\rho_{xy|E}$ between x and y, conditional on the event E can be formulated as,

$$\rho_{xy|E} = \rho_{xy}\left(\rho_{xy}^2 + (1 - \rho_{xy}^2)\frac{Var(x)}{Var(x|E)}\right)^{-1/2} \tag{2}$$

where $Var(x|E)$ is the conditional variance term for x. The details are given in Appendix A. One can see from this equation that unless variables x and y are independent ($\rho_{xy} = 0$) or perfectly correlated ($|\rho_{xy}| = 1$), the correlation coefficient $\rho_{xy|E}$ will be an increasing function of the conditional variance term for x. Regarding the implication of this result on the optimal hedge position chosen, the following theorem holds:[2]

Theorem 1 *Let the returns on the spot (r_p) and futures contracts (r_f) be bivariate normal with variances σ_p^2 and σ_f^2, respectively, and unconditional correlation coefficient ρ_{pf}, $|\rho_{pf}| \leq 1$. Consider any event $r_p \in E$, where $E \subset \Re$ such that $0 < Pr(E) < 1$. The conditional minimum-variance hedge ratio h_E conditional on the event E, is equal to*

$$h_E = hk\Big[(1 - \rho_{pf}^2) + k\rho_{pf}^2\Big]^{-1} \tag{3}$$

where $k = Var(r_p|r_p \in E)/Var(r_p)$ and h is the unconditional optimal hedge ratio.

Extending Theorem (1) to optimal hedging leads to the following corollary.

Corollary 1 *The hedger who fails to take the relation between volatility and correlation into account will under-hedge for those values of k such that $k < 1$.*

Similar to the argument we made for conditional correlation, the conditional minimum variance hedge ratio h_E will be a direct function of the conditional volatility estimate for spot returns. This is a major concern for risk management as the estimation period for the parameters that determine optimal hedge positions should be representative of the period in which the hedge will be used. If the risk manager's goal is to avoid risks during a period of unusual volatility or during an extreme market (however he defines it), the hedge ratio that he estimates from a non-representative estimation period will be suboptimal as he will be using h rather than h_E. Note that $h_E = h$ if $k = 1$, i.e., the event has no impact on volatility.

Having identified the optimal position in the futures contract, one is then interested in how the hedge will perform in minimizing risk. One measure for the degree of hedging effectiveness, HE, is the percentage reduction in the variance of the naked spot price changes calculated as

$$HE = \frac{Var(r_p) - Var(r_{HP})}{Var(r_p)} = 1 - \frac{Var(r_{HP})}{Var(r_p)} = \rho_{pf}^2$$

where HP denotes the hedge portfolio evaluated at the optimal hedge ratio given in equation (1). (See Hull 1998, pg: 92 for details). As can be seen in the above formula, the effectiveness of the hedge is directly linked to how well the returns from the futures contract track the returns from the spot position. So, once again hedge effectiveness is a function of the conditional correlation coefficient between spot and futures returns.

[2] We provide the proof of this theorem in Appendix A.

2.3 Hedging downside risk

Even though variance has long been accepted as a standard measure of risk, the concept of one-sided risk has recently attracted much attention. Recent evidence suggests that corporate managers are mostly concerned about downside risk or the risk of falling below a target rate of return [see Adams and Montesi (1995)]. In fact, the idea was already introduced to the literature by the name lower partial moments [Mao (1970), Lee and Rao (1988)] and downside risk is just a special case of the approach presented in those papers. In the lower partial moment (LPM) approach, the investor first picks a target rate of return, c, and then calculates a probability weighted power function of the shortfall from this target return. Consider a random portfolio return R with distribution function F. The n-th order lower partial moment of R, $L(c, n, R)$ is defined as,

$$L(c, n, R) = E[\max(0, c - R)]^n = \int_{-\infty}^{c} (c - R)^n dF(r) \tag{4}$$

where n is a non-negative integer. A special case of the lower partial moment when $n = 1$ is called the conditional Value at Risk (CVaR) statistic; however in general, one is also interested in the values of the lower partial moment function for $n = 2$, i.e. the *variance* over the undesirable portion of the return distribution, $(-\infty, c)$. Note that an investor who expects to get a higher return from his investment would pick a large c value. Typically, for a hedger who aims to avoid his exposure to adverse movements in spot prices, c values chosen will be negative. However, increasingly positive target rates would imply that the investor uses futures contracts for speculation rather than hedging. Similarly, a large n value indicates that the investor is more concerned about extreme shortfalls. The optimal hedge ratio is then calculated by minimizing the LPM function for a given (c, n) pair. Lien and Tse (1998, 2000) apply the LPM methodology to the case of futures hedges.

In order to estimate the minimum LPM hedge ratios, we use the empirical distribution function. In the case of short hedger, the lower partial moment is estimated by

$$\tilde{L}(c, n, r_s) = \sum_{r_{si} \leq c} (1/N)(c - r_{si})^n$$

where $r_s = r_p - h_s r_f$ is the hedge portfolio return for the short hedger and r_{si} is the i-th sample of the portfolio return. Similarly, the lower partial moment for the long hedger is estimated by

$$\tilde{L}(c, n, r_l) = \sum_{r_{li} \leq c} (1/N)(c - r_{li})^n$$

where $r_l = -r_p + h_l r_f$ is the hedge portfolio return for the long hedger and r_{li} is the i-th sample of the portfolio return.

Once an optimal hedge ratio is obtained, the effectiveness of the futures market in reducing downside risk is calculated by the function,

$$HE = 1 - \sqrt[n]{\frac{L(c, n, r^*)}{L(c, n, r^0)}} \tag{5}$$

where $L(c, n, r^0)$ denotes the lower partial moment obtained when there is no futures hedging, i.e., $h = 0$, and $L(c, n, r^*)$ denotes the minimum lower partial moment value at the optimal hedge ratio for the investor, $h = h^*$. Note that r^0 and r^* denote random returns from the hedge portfolio ($r_p + hr_f$) for $h = 0$ and h^* respectively where r_p and r_f are the returns on the spot and futures positions respectively. The intuition for this measure of hedge effectiveness is provided in Section 4.4.

3 Extreme correlations of spot and futures returns

A major issue of concern in financial risk management is the existence of low probability events in the tails of asset price distributions. Poor management of exposure to market risks and, in some cases, underestimating the consequences of low probability events have cost billions of dollars to a number of large financial institutions in the past (e.g. Long Term Capital Management, Orange County, Metallgesellschaft, Sumitomo Corporation among others). Therefore, for a risk manager who uses futures contracts as hedging instruments, it is crucial to know how returns on assets in the hedge portfolio are correlated during extreme market conditions. An extreme market condition can be due to a major default (e.g. collapse of the Russian economy in the summer of 1998 and its impact on LTCM), severe shortage in supply (in the case of commodity markets), or simply a speculative bubble that bursts. Whatever the cause for an extreme market may be, risk manager is concerned that the hedge will be useless during such periods. The previous section showed that a particular conditioning rule on asset returns (bear, bull or an extreme market) may be critical in the calculation of optimal hedge positions. In this paper, we are interested in the tail behavior of spot and futures returns and its implications on the optimal hedge positions as well as effectiveness of these hedges. Therefore, we start with testing for the existence of correlation asymmetry in futures markets. Then we study the impacts of asymmetry on optimal hedge positions (and resulting hedge performances) for short and long hedgers who aim to minimize downside risks.

The first step in the analysis is to model the dependence structure between spot and futures returns in order to be able to calculate optimal hedge ratios that minimize a given measure of risk. We use a special case of multivariate extreme value theory, i.e., the bivariate version for returns on the spot and futures contracts. As mentioned earlier, the value of diversification will be overstated by those investors who do not take asymmetric correlations (in particular, the increase in downside correlations) into account. Similarly, the hedge positions chosen by risk managers who do not take correlation asymmetry into account will be suboptimal, diminishing the effectiveness of these hedges. The remainder of this section presents a brief description of extreme value theory for univariate and multivariate return distributions. We then explain the estimation procedure.

3.1 Modeling the tails of univariate distributions

Extreme value theory (EVT) studies the tail behavior of distributions of random variables and is now a well-known tool to explain extreme behavior of financial returns over specified investment horizons. Similar to the well-known central limit theorem that provides statistical inferences on the sum of random variables, EVT formulates asymptotic distributions for the maxima or minima of random variables of interest. According to this theory, as long as certain convergence conditions hold, the form of the distribution of extreme returns is precisely known and independent of the underlying return generating process. However, the values of the distribution parameters depend on the underlying distribution. A seminal paper on this topic is Longin (1996). The traditional extreme value theory is divided into two general classes. The first class (*peaks over threshold* models) focuses on those cases in the data that exceed a high threshold whereas the second class (*block maxima* models) divides the data into non-overlapping blocks and focuses on the maxima (or minima) within those blocks. The peaks over threshold models suggest the generalized Pareto distribution which nests the Pareto, uniform, and exponential distributions as limiting distributions that explain tail behavior [see Balkema and de Haan (1975) and Pickands (1975)]. The block maxima models suggest the generalized extreme value distribution proposed by Jenkinson (1955) and include three standard extreme value distributions obtained by Gnedenko (1943): Frechet, Weibull, and Gumbel. Our methodology is based on the peaks over threshold method that uses the generalized Pareto distribution suggested by Pickands (1975). Now we provide a brief description of the theory for univariate distributions.

Define R as the return on a security and F_R as its distribution function. We define extreme returns as those that lie beyond a threshold value θ. A return R is higher than θ with probability p where p is obviously a function of the threshold value chosen, i.e., $p = 1 - F_R(\theta)$. If one has perfect knowledge of the underlying return distribution F_R, then the cumulative distribution F_R^θ of extreme returns i.e., those beyond the threshold value, can be formulated as $F_R^\theta(r) = [F_R(r) - F_R(\theta)]/[1 - F_R(\theta)]$ for $r > \theta$. (For simplicity, we focus on the right tail of the distribution of returns ($R > \theta$), however extension to the lower tail can easily be done by assuming negative values for θ) However, this is a difficult task in most financial applications as the exact distribution of returns is not known. Therefore, one resorts to analyzing the asymptotic behavior of return exceedances, i.e., a distribution function G_R^θ that approximates F_R^θ. Pickands (1975) suggests the generalized Pareto distribution given by

$$G_R^\theta(r) = 1 - \max\Big[0, 1 + \xi\frac{(r-\theta)}{\sigma}\Big]^{-\frac{1}{\xi}} \tag{6}$$

where σ is the dispersion parameter that is a function of the threshold value θ and the return distribution F, and ξ is the tail index. The generalized Pareto distribution in (6) nests three distribution functions, i.e., Pareto, uniform and the standard exponential distribution:

Pareto Distribution: $G_R^\theta(x) = 1 - x^{-1/\xi}$ for $x \geq 1$, $\xi \neq 0$,
Uniform Distribution: $G_R^\theta(x) = 1 - (-x)^{1/\xi}$ for $x \in [-1, 0]$, $\xi \neq 0$,
Exponential Distribution: $G_R^\theta(x) = 1 - e^{-x}$ for $x \geq 0$, $\xi = 0$

where x refers to the transformed maxima (or minima) so that the limit distribution is a non-degenerate one. [See Pickands (1975) for further detail]

The tail index ξ, also called the shape parameter, defines the tail of the distribution of returns. The parameter $\xi > 0$ indicates a polynomially declining tail (Pareto) which is the case for the fat-tailed distributions found in financial returns. The parameter $\xi < 0$ corresponds to distributions with no tail (truncated distributions) and $\xi = 0$ indicates an exponentially decreasing tail, the case for thin-tailed distributions. In short, the *peaks over threshold* approach in extreme value theory can be used to model return exceedances and in the case of univariate distributions, it holds that return exceedances can only converge toward a generalized Pareto distribution which nests the three general forms of distributions stated above. The advantage of using the limiting distribution function G_R^θ is that no detailed knowledge of F_R is needed.[3]

3.2 Modeling the dependence of spot and futures returns

One issue of concern for the portfolio manager is how the returns on assets in the portfolio are correlated during such periods when he needs diversification most. These may be periods of high volatility, extreme market conditions (however one defines it), or simply a bear market. Similarly, a risk manager who uses futures contracts as hedging instruments is interested to know how returns on assets in the hedge portfolio are correlated during extreme market conditions. As mentioned earlier, optimal hedge positions as well as effectiveness of hedges depend on how well returns on the futures contract track returns on the spot.

In this paper, we use multivariate extreme value theory to model extreme correlations between spot and futures returns across ten different markets. Ledford and Tawn (1996) present a general framework for the multivariate case, we deal with the bivariate case. Consider $R = (R_p, R_f)$ the return vector and $\theta = (\theta_p, \theta_f)$ the threshold vector for spot and futures returns respectively. Our goal is to fit a bivariate version of the generalized Pareto distribution, G_R^θ, described in the previous section. Ledford and Tawn (1996) show that G_R^θ must satisfy the following two conditions:

1. The univariate marginal distributions $G_{R_p}^{\theta_p}$ and $G_{R_f}^{\theta_f}$ for spot and futures returns respectively are also generalized Pareto distributions.

2. There exists a function, D, called the dependence function, $D : \Re^2 \rightarrow \Re$ such that

$$G_R^\theta(r_p, r_f) = \exp\Big[-D\Big(-\frac{1}{log(G_{R_p}^{\theta_p})(r_p)}, -\frac{1}{log(G_{R_f}^{\theta_f})(r_f)}\Big)\Big]. \quad (7)$$

[3] See Jansen and de Vries (1991) for further detail.

So, unlike the univariate case, the bivariate asymptotic distribution is not completely specified as the shape of D is not known. Longin and Solnik (2001) use the commonly used logistic function of Gumbel (1961) for D given by

$$D(y_p, y_f) = (y_p^{-1/\alpha} + y_f^{-1/\alpha})^{\alpha} \tag{8}$$

where $y_i = -1/logG_{R_i}^{\theta_i}(r_i)$ for $i = p, f$. The parameter α describes the level of dependence between extreme returns for the two series [see also Tawn (1988)]. Tiago de Oliveira (1962) shows that correlation coefficient ρ in the bivariate case is related to α as $\rho = 1-\alpha^2$. Special cases are asymptotic independence, $\alpha = 1$, and total dependence, $\alpha = 0$.

In short, the multivariate version of extreme value theory shows that the distribution of extreme returns can only converge toward a distribution characterized by generalized Pareto marginal distributions and a dependence function. So, the first step is to estimate this dependence function and test whether the correlation of extreme returns is equal to zero. The next step is to analyze the implications of extreme correlations on optimal hedge ratios as well as hedge performances.

3.3 Inference for the model

This section aims to explain how the tails of univariate distributions of spot and futures returns are estimated. It then explains the estimation procedure for extreme correlations between spot and futures returns. The theoretical details for generalized multivariate distributions are given in Ledford and Tawn (1996). From now on, we will denote $F_{R_p}^{\theta_p}$ as F_p, $F_{R_f}^{\theta_f}$ as F_f, and F_R^{θ} as F. We will use the same simplified notation for G too.

Univariate Marginal Distributions: The tail of the distribution of each return R_i denoted by F_i [$i = p$ (spot), f (futures)] is estimated using the following functional form suggested by Davison and Smith (1990) and Ledford and Tawn (1996),

$$F_i(r_i) = (1 - p_i) + p_i \cdot G_i(r_i) = 1 - p_i \cdot \max\left[0, 1 + \xi_i \frac{(r_i - \theta_i)}{\sigma_i}\right]^{-1/\xi_i} \tag{9}$$

for a given threshold value θ_i ($i = p, f$). Note that the function is defined for $r_i > \theta_i$ (for the right tail). Thus, the model in (9) suggests that a marginal observation does not belong to the tail with probability $1-p_i$ and is drawn from the limiting univariate distribution G_i with probability p_i. The rationale behind the model is that, for a marginal observation which fails to exceed the threshold, the only relevant information it contains is that it occurs below the threshold, not its actual value. As will be shown in the maximum likelihood procedure, this rationale determines the form of maximum likelihood function constructed.

Bivariate Distribution and Extreme Correlations: One approach to model spot and futures return dependencies under different conditioning scenarios (e.g. bear, bull, or an extreme market) is through exceedance correlations of Longin and Solnik (2001). Other approaches include asymmetric GARCH-M [Engle and Kroner (1995), Bekaert and Harvey

(1997), De Santis, Gerard and Hillion (1999), and Bekaert and Wu (2000)], bivariate normal with Poisson jumps, [Das and Uppal (2001)], regime-switching bivariate normal [Ang and Bekaert (2000)], and regime switching GARCH [Gray (1996)]. We follow Ang and Chen (2002), Longin and Solnik (2001), and Silvapulle and Granger (2001) and focus on exceedance correlations. Exceedance correlations between spot and futures returns are relevant to our analysis since the second part of the article is concerned with the effectiveness of futures markets in minimizing downside risk beyond different target returns selected. Given a threshold level, θ, an exceedance correlation is a measure of dependence between two variables conditioned on the fact that these variables have taken values beyond the threshold chosen. Exceedance correlations are estimated separately for upside and downside movements beyond the threshold. Define $\tilde{R}_p$ and $\tilde{R}_f$ as standardized returns for spot and futures contracts respectively. The exceedance correlation at an exceedance level θ is given by

$$\rho^e(\theta) = \begin{cases} corr(\tilde{R}_p, \tilde{R}_f \mid \tilde{R}_p > \theta; \tilde{R}_f > \theta) & \text{if } \theta \geq 0 \\ corr(\tilde{R}_p, \tilde{R}_f \mid \tilde{R}_p < \theta; \tilde{R}_f < \theta) & \text{if } \theta \leq 0 \end{cases} \tag{10}$$

One problem with exceedance correlations estimated using the formula in (10) is the conditioning (or selection) bias that is due to splitting the sample according to realized values alone. For example, if we generate pairs of bivariate normal random variables with an unconditional correlation coefficient $\overline{\rho}$ and calculate the exceedance correlations according to the formula in (10), the values of conditional correlations $\rho^e(\theta)$ estimated will be different than $\overline{\rho}$ even though the data are generated from a bivariate normal distribution with correlation coefficient $\overline{\rho}$. One example for this is explained in Appendix A where we prove equation (2). Boyer, Gibson and Loretan (1999) provide analytical explanations as to how different conditioning rules impact the correlation values calculated.[4] Ang and Chen (2002) provide the closed-form solution to calculate conditional correlations. There are several alternative ways to calculate exceedance correlations. One approach is suggested in Ang and Chen (2002) where they develop a summary statistic to quantify the degree of asymmetry in correlations after correcting for conditioning biases. The advantage of their approach is that it is not model specific, i.e., the degree of asymmetry can be estimated relative to any other given distribution, not only the bivariate normal. An alternative approach is recently suggested by Campbell, Kedjik, and Kofman (2002) in which conditional quantile correlations can be estimated within a Value at Risk framework. The advantage of this methodology is that it does not suffer from the conditioning bias and allows for a direct comparison of the conditional quantile correlation with the unconditional correlation.

The methodology applied in this paper originates from Ledford and Tawn (1996) and is based on extreme value theory. Longin and Solnik (2001) apply the same methodology on international stock indices. Having defined univariate marginal distributions for spot and futures returns as in equation (9) and the bivariate distribution as in (7), we estimate the dependence function in (8) using the maximum likelihood criteria. Given the threshold values (θ_p, θ_f) for spot and futures returns respectively, we split the sample into four regions

[4] A more comprehensive analysis of extreme dependences can be found in Malevergne and Sornette (2002).

depending on the return for each contract exceeding its threshold or not. For a given marginal observation $(r_{pt},\ r_{ft})$ on day t, the likelihood contribution $L(r_{pt}, r_{ft})$ is given by

$$L(r_{pt}, r_{ft}) = \begin{cases} F(r_{pt}, r_{ft}) & \text{if } r_{pt} < \theta_p, r_{ft} < \theta_f \\ \frac{\partial F(r_{pt}, r_{ft})}{\partial r_f} & \text{if } r_{pt} < \theta_p, r_{ft} > \theta_f \\ \frac{\partial F(r_{pt}, r_{ft})}{\partial r_p} & \text{if } r_{pt} > \theta_p, r_{ft} < \theta_f \\ \frac{\partial^2 F(r_{pt}, r_{ft})}{\partial r_p \partial r_f} & \text{if } r_{pt} > \theta_p, r_{ft} > \theta_f \end{cases}$$

where F denotes the bivariate distribution of return exceedances for the futures and spot contracts. Note that each function in the above formula is evaluated at the value $\max(r_{it}, \theta_i)$ for $i = p, f$. So, for observations below the threshold, the threshold value is used to evaluate the function, otherwise the observation value is used. Having defined the likelihood contribution for a marginal observation we then calculate the likelihood for the full sample of T independent observations of returns by

$$L[\{(r_{p1}, r_{f1}), \ldots, (r_{pT}, r_{fT})\}|\Phi] = \prod_{t=1}^{T} L(r_{pt}, r_{ft})$$

where $\Phi = \{p_p, p_f, \sigma_p, \sigma_f, \xi_p, \xi_f, \alpha\}$ is the set of parameters to be estimated.[5] Note that the parameter α in the dependence function D in (8) is related to the exceedance correlation value ρ^e by $\rho^e = 1 - \alpha^2$. Appendix B provides the mathematical details of the maximum likelihood procedure. Further detail and generalization to multivariate case is given in Ledford and Tawn (1996).

4 Empirical Findings

4.1 Data

The data set consists of ten futures contracts: three currency futures contracts (British Pound, Deutsche mark and Japanese Yen), five commodity futures contracts (soybean oil, wheat, crude oil, corn and cotton) and two stock index futures contracts (NYSE composite and S&P 500). Daily spot and nearby futures prices are obtained from Commodity System, Inc. The sample period extends from January 1988 to June 1998. Nearby futures prices are constructed with contract rollover occurring about one week before the maturity in most cases. The trading volume is used as a criterion in deciding the actual rollover date. Daily returns are calculated as the differenced logarithmic prices and returns at the rollover dates have been calculated over the same contract. The total number of observations for each series varies from 2631 to 2659. Table 1 presents some summary statistics for futures and spot returns.

[5] The maximum likelihood procedure assumes that return pairs are serially independent.

The two stock indices and their associated futures have the highest average returns at a daily average ranging between 0.041% and 0.055%, however these contracts do not have the highest volatility. We observe mixed signs in the average returns for commodities. Wheat and crude oil futures have the largest expected returns among commodities with 0.020% and 0.024% for wheat and crude oil respectively. All three foreign currencies experienced negative average spot and futures returns during the same period with the exception of British pound futures. On the other hand, currency returns are the least volatile, followed by stock indices. Commodity markets are the most volatile with crude oil experiencing a standard deviation of 2.488% and 2.252% in spot and futures markets respectively.

Table 1 also shows that there is no serial correlation in each return series. Spot and futures returns are highly correlated with sample correlation values larger than 0.95, in currency and stock index markets. The correlation values range from 0.565 (wheat) to 0.951 (soybean oil) among commodities. The smaller correlation values observed between spot and futures returns in commodity markets may depress the usefulness of futures contracts as a risk management instrument.

4.2 Spot and futures return correlations

Table 2 gives the maximum likelihood estimates of the parameters of the bivariate distribution of spot and futures return exceedances evaluated at different threshold values θ. Additionally, Figure 1 provides the plots of exceedance correlations between futures and spot returns at selected thresholds. Threshold values are defined in terms of standard deviations away from the mean of each return series. For example, in the case of S&P 500 spot returns, we define the threshold values as $\theta_p = \mu_p \mp i\sigma_p$ for $i = 3, 2, 1, 0$ where μ_p is the mean, and σ_p is the sample standard deviation of spot returns for S&P 500. Note that for $i = -3, -2, -1, 0$, we calculate exceedance correlations on the downside and for $i = 3, 2, 1, 0$, we calculate exceedance correlations on the upside. Panels A and B in Table 2 provide the estimates for negative and positive return exceedances respectively. Threshold values for futures returns are defined in a similar way. Note that threshold values are different for each contract as they differ in their mean and standard deviation estimates. For example, the threshold $\theta = -2$ corresponds to -1.763% for the left tail and 1.850% for the right tail in the case of S&P 500 futures. The selection of threshold values is constrained by the fact that there are very few observations beyond $\mp 3\sigma$ away from the empirical mean. In the worst case, we use 17 observations to estimate model parameters. The same logic is used when we set target rates of return in the lower partial moment function.

In Figure 1, we also provide plots of the theoretical exceedance correlation values from a bivariate normal distribution with the same mean, standard deviation and unconditional correlation values estimated from the sample. Exceedance correlations for the bivariate normal are estimated through simulation with mean, standard deviation and correlation values equal to the estimates from the sample of spot and futures returns associated with the particular contract. Under bivariate normality, the plot of exceedance correlations is expected to have a tent shape meaning correlations would symmetrically decrease to zero as one moves towards the tails of the distribution.

Regarding the parameter estimates in Table 2, we observe, in general, high standard deviations for estimates obtained at thresholds $\theta = -3$ and $+3$. This is a very common observation for studies that use extreme value theory and is most likely due to the limited number of observations used at extreme thresholds. The tail probability estimates p_i are in most cases very close to the empirical probability of returns exceeding the corresponding thresholds. For example, in the case of S&P 500 contracts, the estimated left tail probability (p_p) for spot returns beyond two standard deviations away from its mean ($\theta = -2$) is equal to 2.67% with a standard error of 0.006 whereas, over the period January 1988 to June 1998, there are 72 out of 2644 daily returns to the left of two standard deviations away from the mean, leading to an empirical frequency of 2.72%. We observe similar accuracy in tail probabilities across all contracts analyzed.

In the case of commodities contracts, except for Crude Oil and Wheat, the findings do not indicate that correlations are significantly higher as we move towards extreme return values. We also do not observe any asymmetry regarding up and down markets. However, we observe a different pattern for Crude Oil and Wheat contracts in the sense that spot and futures return correlations are well above what one would expect from a bivariate normal distribution. Furthermore, we find that correlations increase significantly on the downside, i.e. when negative thresholds are selected, and the degree of covariation gets even stronger as we move towards extreme negative returns. One major difference between Crude Oil and Wheat contracts is that standard deviations for Crude Oil spot and futures returns also increase significantly as increasingly negative thresholds are selected.

Regarding foreign exchange contracts, the data does not provide support for correlation asymmetry. Spot and futures return correlations do not seem to increase significantly as we move towards extreme returns, and we do not also observe an asymmetric pattern for downside and upside correlations. Regarding standard deviations of futures and spot returns, Japanese Yen seems to be different in the sense that we observe higher correlations at the tails. However, regarding stock index contracts, our findings indicate strong support for correlation asymmetry. Examining the plots for NYSE 100 and S&P 500 contracts in Figure **??**, one can see that correlations are greater on the downside than on the upside, furthermore correlations increase as we move towards the extremes. We also observe an interesting pattern in volatility estimates as well. Examining the estimates of dispersion parameters (σ) for NYSE 100 and S&P 500 contracts in Table 2, we see that volatility estimates obtained for negative return exceedances are almost always greater than those for positive return exceedances indicating higher volatility on the downside. In fact this finding conforms with the empirical standard deviation estimates. These findings indicate that although volatility increases substantially towards increasing thresholds on the downside, covariance increases at a higher rate so that we observe higher exceedance correlations. The implications of this observed asymmetry on optimal hedge positions and hedge performances are explained next.

4.3 Optimal hedge ratios for short and long hedgers

Before we provide our findings on optimal hedge positions for short and long hedgers, a few comments are in order. Regarding the different types of contracts analyzed, one would be interested in knowing what type of hedger will be affected by asymmetric correlations. For a hedger taking positions in a stock index futures contract, one might suggest that the motivation would most likely stem from a *portfolio immunization* strategy. In this strategy, the fund manager whose goal is to protect the value of his fund from going below a certain dollar value, takes a short position in a corresponding index futures such that portfolio losses due to downside market moves will be somewhat eliminated by gains in the futures position. Therefore, regarding a portfolio immunization strategy, one would be interested in the findings that we obtain for a short hedger in an index futures. However, regarding commodities and foreign exchange futures, one might argue both ways depending on which side of the transaction the hedger is. For example, a wheat farmer will be interested in our results for a short hedger whereas a flour factory that needs to purchase wheat would be interested in our results for a long hedger. The same logic applies to foreign exchange contracts depending on whether the investor will be receiving the foreign currency or making a payment in terms of foreign currency in the future. So, our results would be of interest to both types of hedgers in these contracts.

Having found mixed evidence on asymmetry in correlations between the returns on futures and spot contracts, we then examine the implications of asymmetry on the optimal hedge ratios and hedging performances for short and long hedgers. Several properties of optimal hedge ratios with respect to the target rate c and order of moment n are provided in Appendix C. We also provide in Appendix **??** several properties of the minimum LPM hedge ratios under the assumption of normality. In theory, asymmetry should have two broad implications on optimal hedge positions. First implication will be on the size of optimal positions. Since correlations tend to be greater for extreme moves, we should expect smaller optimal hedge positions estimated, as higher correlations would enable the same amount of hedging effectiveness with smaller futures positions taken. Second, since correlations tend to be higher on the downside than on the upside, one type of hedger (short or long) should find better hedging potential in such market conditions. This means that asymmetry should help either the long or the short hedger minimize the same amount of downside risk as the other by taking smaller positions in the futures contract.

From a hedger's perspective, the main use of futures contracts for hedgers is to avoid exposure to adverse movements in spot prices. This means the target rates of return (c) chosen by hedgers will be typically negative. However, increasingly positive target rates would imply that the investor uses futures contracts for speculation rather than hedging. Consistent with the definition of the threshold levels used to estimate exceedance correlations in section 4.2, we set the target rates in units of standard deviations of spot returns for each contract separately. Since the hedger's primary goal is to avoid the exposure to adverse movements in the spot market, we consider a list of possible target rates in the general form, $c_i = \mu_p + i\sigma_p$ for $i = -3, -2, -1.5, -1, -0.5, 0$ where μ_p is the mean, and σ_p is the standard deviation of spot returns for the market analyzed. We also use two positive target rates, i.e. $i = +0.5$

and +1 (in terms of multiples of σ_p), in order to see how optimal hedge positions behave on the positive target territory. Note that the target rates will be different for each market as the standard deviations of the spot returns for each market will be different. The estimated optimal hedge ratios are reported in Table 3. Also note that the numbers in the target return column of the table indicate the units of standard deviations away from the mean. For example, the value of -3 indicates that the target rate used for that case is $(\mu_p - 3\sigma_p)$ where subscript p denotes the spot contract for that market.

Estimated optimal hedge ratios, in general, conform with the predictions stated earlier. In Appendix C, we show that the intuition approach to explain how optimal hedge ratios should behave with respect to the target rates does not always work. The results in the appendix indicate that the behavior of optimal hedge ratios with respect to target rates mainly depends on the sign of expected futures returns and covariance between futures and spot returns in the subsample of data where $c - r_p + h_s r_f > 0$ is satisfied. For the long hedger however, a similar argument can be made under the condition $c + r_p - h_l r_f > 0$. The plots in Figure **??** indicate several patterns in the estimates of optimal hedge ratios. First, we discuss the case in which $c = 0$, one of the conventionally adopted target returns. In each asset, we observe similar h_s^* and h_l^* values for $n = 1$ and $n = 2$. Exception to this is Cotton contracts where we observe higher hedge positions when $n = 1$. This indicates that short and long hedgers will assume smaller futures positions as they assign higher weights on bad outcomes.

We do not observe a systematic pattern for the size of the optimal hedge ratios with respect to the order of moment (n). Similar to the results in Appendix C, one can show that the sign of $(\partial h^*/\partial n)$ depends on the sign of expected futures return and covariance in the portion of return distribution where shortfalls are nonnegative. Therefore, this finding may be due to the variance-covariance asymmetry reported earlier. At zero threshold, when $n = 1$, hedge ratios for both short and long positions are closer to one than the minimum variance hedge ratio. In other words, a hedger who cares for minimum expected shortfalls is more likely to assume an equal but opposite position in the futures market against the spot position. In case that $n = 2$, as expected, the minimum variance hedge ratio lies in between the two hedge ratios.

Regarding the differences between contracts across the target rates selected, optimal hedge ratios for commodities in general indicate a different pattern compared to foreign exchanges as well as stock indices. For short hedgers in commodity contracts, we observe a general downward sloping pattern in optimal hedge ratios with a minimum hedge position at the target value of zero, and increasing hedge positions as we choose higher target rates beyond zero. This indicates that a short hedger takes smaller positions in the corresponding commodity futures as higher target rates are selected. One reason for this may be that hedge performances decrease in an exponential manner as higher target rates are selected, therefore futures contracts lose importance at these target rates leading to smaller optimal hedge positions. Findings for long hedgers indicate a less consistent pattern however. Optimal hedge ratios for Soybean Oil, Cotton and Corn increase in general as higher target rates are selected. However, hedge ratios for Wheat and Crude Oil are relatively stable for different target rates.

In the case of foreign exchange and stock index contracts, we observe a quite different pattern. The plots in Figure **??** indicate that as smaller target rates are chosen, optimal hedge positions also decrease as well. Furthermore, we observe similar hedge ratios at different n values when negative target rates are selected. As stated in the appendix, when the target rate is sufficiently negative, we find that the same optimal hedge ratios minimize the LPM function at different n values. Note that these findings correspond to cases where we observe perfect hedge performance, i.e. $HE = 1$, in currencies and stock indices. Another observation is a decline in the optimal hedge ratios for both hedgers when target returns are sufficiently negative. This indicates that stock indices and currencies allow both type of hedgers to minimize all downside risk (for sufficiently negative target returns) by taking smaller positions in the futures markets. This finding is most likely related to the high correlation values between spot and futures returns for these contracts. In the next section, we will show that hedge performances approach to a perfect hedge at smaller target rates (on the downside) and it will be clear that there is a link between the absolute values of optimal hedge positions and performances of these hedges.

Regarding stock index contracts, when we allow for arbitrary target returns, we find that the optimal hedge ratios for both type of hedgers decrease as smaller target rates are selected - this is similar to what we observe for foreign exchanges. However, in the positive target rate region, the findings are quite different. Unlike the case for negative target rates, we find that optimal hedge ratios for the long hedger increase with the target rate. This is probably an implication of correlation asymmetry. The fact that correlations tend to be greater on the downside leads this type of hedger to take on greater futures positions in order to maintain higher target rates on the upside. To summarize, the most prominent pattern observed is a decreasing futures position for both type of hedgers in response to a decrease in the target rate. In other words, as the hedger is satisfied with smaller target rates, he will assume a smaller futures position. However, on the upside, i.e. for positive target rates, long hedgers who expect larger returns, will become more active in futures trading.

Comparing the size of the optimal futures positions for short versus long hedgers, we find that short hedgers in Soybean Oil, Crude Oil and Corn, need to take on higher positions in the futures contract and this holds in general for all target rates. However, in the case of British Pound, Japanese Yen and stock index contracts, we observe higher optimal hedge positions for short hedgers only when negative target rates are selected. However, as higher target rates are set, optimal positions for short hedgers tend to be smaller than those for long hedgers. This conforms with our prediction that long hedgers will benefit from futures trading for increasingly negative target rates, as higher correlations on the downside will allow them to achieve the same amount of hedging performance by taking smaller positions. Next we analyze the performance of these hedges.

4.4 Hedge Performances

As noted earlier, asymmetry in correlations between the spot and futures contracts should lead one type of investor to benefit from these markets more than the other. One expects that for markets where higher correlations are observed on the downside, this should benefit long hedgers more as these investors would enjoy the high correlation between their spot holdings and the corresponding futures contracts. In order to evaluate the effectiveness of the futures market in reducing the downside risk, we consider the hedging performance measure given in equation (5). For the short hedger, the LPM function is

$$L(c, n, r) = E[\max(0, c - r_p + h_s r_f)]^n \tag{11}$$

where h_s is the hedge ratio chosen by the short hedger. So, the hedging performance function (HE) we use is actually a function of the ratio of the LPM value evaluated at the optimal hedge ratio to the LPM value obtained when there is no hedging. Similar to the minimum-variance hedge in which hedge effectiveness is measured through percentage reduction in variance, HE measures percentage reduction in the LPM function specified at a particular (c, n) combination. Note that when the futures contract completely eliminates the downside risk, $HE = 1$, and if the futures contract does not help at all, we will obtain $HE = 0$. So, the hedge performance value will range from 0 to 1 such that a larger number indicates better hedging performance. In order to make the point clear, consider the case $c = -0.5\%$ and $n = 1$, i.e. the hedger aims to minimize the expected loss beyond a target return of -0.5%. $HE = 1$ would indicate that, at the minimum LPM hedge ratio, the hedge portfolio does not lose more than 0.5%, i. e. no worse than 0.5% loss is guaranteed at the corresponding hedge ratio. This is the case where the hedger eliminates all downside risk beyond the specified target return for the hedge portfolio. Note that in the case of variance minimization, a perfect hedge is possible when there is perfect correlation between spot and futures returns, i. e. $\rho_{pf} = 1$. However, perfect correlation is not necessarily required to obtain $HE = 1$ when the LPM function is minimized. Regarding the long hedger, we apply a similar procedure, however this time defining the LPM function as

$$L(c, n, r) = E[\max(0, c + r_p - h_l r_f)]^n \tag{12}$$

where h_l is the hedge ratio chosen by the long hedger. In empirical studies, the lower partial moment is not observable. Instead, the estimates derived from empirical distribution functions are substituted into equations (11) and (12). The numbers in parentheses in Table 3 are the hedging effectiveness values for S&P 500 contracts.

Figure **??** provides the plots of hedge performances for short and long hedgers at selected thresholds. The effect of asymmetric correlations is easier to observe in the case of hedging performances. In the case of stock index contracts where we have earlier reported asymmetry in correlations, we observe - for both n values - that the hedging performance decreases with increasing target returns for both short and long hedgers. Thus, as the hedger aims for higher target rates anticipating a larger return, the futures contracts provide less benefit. Note that selecting higher target rates might also indicate speculative motives rather than hedging. Considering negative target rates however, we find that in the case of stock index contracts, when the target return is set to 1.5 standard deviations away from the mean on the downside, futures contracts completely eliminate the downside risk. A similar pattern is observed for foreign exchanges as well, however we believe this is most likely due to high values of correlations obtained for these contracts.

Both short and long hedgers experience a sharp decline in the hedging performance when the target rate is set at positive values. For example regarding S&P 500 contracts, from $c = 0$ to $c = +0.5$, hedge performances for short and long hedgers decrease from 0.658 (short) and 0.657 (long) to 0.185 and 0.245, respectively when $n = 1$. Another observation is that when $c > 0$, hedgers improve their hedging performance as n increases. This means that both type of hedgers find S&P 500 futures contracts to be more useful in reducing downside risk when higher weights are assigned to large shortfalls from the target rate.

Comparing the hedging performance between short and long hedgers, we find that hedge performances for long hedgers are in general larger than those for short hedgers. That is, the long hedger is able to reduce more downside risk than the short hedger. However, although the differences in hedging performance are small in the case of negative target rates, the difference can be large if c is sufficiently high. In short, we find that the long hedger benefits more from futures trading than the short hedger in reducing the downside risk.

4.5 Post-sample analysis: The minimum–variance hedge vs. the minimum–LPM hedge

This section presents our results on how hedges based on minimizing downside risk perform relative to hedges based on the traditional minimum variance criterion. For this purpose, we split the sample into two periods. We used return data from the 1988–1993 period to estimate minimum variance hedge ratios as well as hedge ratios that minimize the LPM function for selected combinations of target return (c) and order of moment (n). We then used these optimal hedge ratios to construct hedge portfolios over the period 1994–1999. Table 4 presents the optimal hedge ratios and summary statistics for the hedge portfolios. Panels A and B in the tables provide our findings for minimum variance and minimum LPM hedges respectively. Our findings generally indicate that hedges based on minimizing downside risk perform in many cases better than hedges based on variance minimization.

For example, regarding S&P 500 contracts again, for the 1994–1999 returns of hedge portfolios, we observe that minimum LPM hedges yield higher average returns compared to the minimum variance hedge, which has an average hedge portfolio return of 0.020%. Higher average returns for minimum LPM hedges coincide with higher standard deviations of hedged portfolios with a maximum of 0.377% when $c = -3$. One observation is that the variability increases as target rates move further away from zero, i.e. as we concentrate on shortfalls on a single tail (rather than both tails in the case of variance minimization) only. Note that the hedge ratios and summary statistics obtained for the minimum variance hedge are quite similar to the hedge ratios and summary statistics obtained for the minimum LPM hedge when the target return is set to zero ($c = 0$).

Regarding the risk-return tradeoff for the hedge portfolios, we also provide the reward-to-risk (RR) ratios (μ/σ) in column 5 of Table 4. One interesting observation is that the RR ratios for the hedge portfolios are almost always higher for minimum LPM hedges than that of the minimum variance hedge (0.102). We find that setting a target rate one standard deviation below the average spot return yields the highest RR ratio around 0.124. Additionally, Table 4 provides the minimum, the maximum and the 1% and 5% percentiles for the hedge portfolios. Our findings suggest that minimum LPM hedge ratios yield generally better results than the traditional minimum variance hedge for the 1994–1999 period. Although variability increases for minimum LPM hedge portfolio returns, it is possible to choose an optimal target return for the LPM function so that a higher reward-to-risk ratio is achieved. In the case of S&P 500 futures, we find that choosing a target rate of one standard deviation below the average spot return yields the best post–estimation sample performance for the hedge portfolio.

5 Conclusion

Reliability of risk parameters is critical to financial analysts who measure and manage market risks. It has long been known that volatility of stock returns at the market and individual firm level is not constant but changes over time. However, recent research on correlations among equity returns suggest the notion of correlation asymmetry. The general finding is that correlations tend to be much greater on the downside, especially for extreme moves, than on the upside. One explanation is that the volatility of asset returns displays asymmetric response to good and bad news. However, it is still not clear whether asymmetry in correlations is due to time variation in volatility or due to a change in the underlying return generating process.

Correlation asymmetry has significant implications for several applications. In the case of portfolio diversification, failing to take into account the increase in downside correlations leads to suboptimal portfolio weights, eroding the benefits from diversification in bear markets when it is most needed. If all stocks tend to fall together when the market falls, the value of diversification may be overstated by the portfolio manager who fails to take this into consideration, leading him to over-invest in risky assets. Similarly, in the case of risk management, risk managers face the possibility that their hedges will be useless when they are most needed, namely during stressful market conditions. Furthermore, asymmetric correlations make it harder for risk managers to decide on what historical data should be used to estimate parameters for worst case analyses and to stress test their models.

Although correlation asymmetry is widely studied in domestic equity markets [e.g. Loretan and English (2000), Silvapulle and Granger (2001), Ang and Chen(2002)] and in international markets [e.g. Ang and Bekaert (2000), Das and Uppal (2001), Longin and Solnik (2001)], it has not yet been extended to derivatives markets. The analysis of correlation asymmetry in derivatives markets is important as correlations are also crucial for pricing and hedging derivative securities with payoffs that depend on more than one asset. In this essay, we fill this gap in the literature. We study the conditional correlation structure of futures market returns across several markets and analyze the implications of asymmetric correlations on optimal hedge positions as well as performance of these hedges. Clearly, this is an important issue for risk managers as unstable correlations make it difficult, perhaps impossible, to hedge exposure to risk factors by taking an offsetting position in another asset.

This study is also original in the sense that it differentiates between short and long hedgers. For a hedger taking positions in a stock index futures contract, one might suggest that the motivation would most likely stem from a portfolio immunization strategy. The fund manager whose goal is to protect the value of his fund from going below a certain dollar value, will be taking a short position in a corresponding index futures such that portfolio losses due to downside market moves will be somewhat eliminated by gains in the futures position. Therefore, regarding a portfolio immunization strategy, one would be interested in the findings that we obtain for a short hedger in an index futures. However, regarding commodities and foreign exchange futures, one might argue both ways depending on which side of the transaction the hedger is. So, our results would be of interest to both types of hedgers in these contracts.

Our analysis of futures and spot returns on a number of contracts over 1989-1999 indicates somewhat conforming results to those obtained from equity markets. We find that correlation asymmetry is not just a phenomenon we observe in domestic and international equity returns, but that it also exists in the futures markets, especially futures contracts based on stock indices. However, we do not find supporting evidence for commodities and foreign exchanges in general. One exception to this is Crude Oil. In the case of Crude Oil and stock index contracts, we find that correlations between spot and futures returns conditional on downside moves tend to be greater, especially for extreme moves, than on upside moves. There are two implications of this result. First, a risk manager who fails to take into account higher downside correlations between the returns on the spot and futures contracts will understate the value of these markets as hedging instruments. So, during bear markets (and also extreme markets) when correlations tend to be higher, optimal hedge ratios chosen by the risk manager will be suboptimal if asymmetry is not taken into account. Second, correlation asymmetry will affect hedgers with short and long positions differently if they are concerned about downside risk. Since, short and long hedgers will be interested in the opposite tails of the return distribution for their hedge portfolios, asymmetric correlations will lead to better hedging performance for one type of hedger. In general, we find that the long hedger benefits more from futures trading than the short hedger in reducing the downside risk. This finding seems to be due to higher downside correlations that provide better hedging opportunity for investors who hold long positions. Regarding optimal hedge positions, we find that optimal hedge positions for the short hedger tend to be smaller than those for the long hedger as higher target rates are set for the hedge portfolio. This conforms with the argument that long hedgers will benefit more from futures trading for negative target rates, as higher correlations on the downside will allow them to achieve the same amount of hedging performance by taking smaller positions.

Finally, our analysis of post sample hedging performances indicates that, in general, optimal hedge ratios that minimize the downside risk have the potential to yield better results than the traditional minimum variance hedge. Although variability increases for minimum Lower Partial Moment (LPM) hedge portfolio returns, it is possible to set an optimal target return for the LPM function so that a higher reward-to-risk ratio is achieved. Although a general rule cannot be suggested in terms of what the specific target rate should be, in the case of S&P 500 futures for example, we find that setting a target rate of one standard deviation below the average spot return yields the best post sample performance for the hedge portfolio.

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A Optimal hedging under conditional correlation

Let the return on the spot contract, r_p, and the return on the futures contract, r_f, be bivariate normal with means μ_p and μ_f and variances σ_p^2 and σ_f^2. Denote the unconditional correlation between r_p and r_f as ρ_{pf}, $|\rho_{pf}| \leq 1$. r_p and r_f can be expressed as a linear function two standard normal random variables u and v as the following:

$$r_p = \mu_p + \sigma_p u \tag{A.1}$$

$$r_f = \mu_f + \rho_{pf}\sigma_f u + \sqrt{1-\rho_{pf}^2}\sigma_f v. \tag{A.2}$$

Let E denote any event such that $0 < \Pr(E) < 1$. The conditional correlation coefficient ρ_E between r_p and r_f is by definition

$$\rho_E = \frac{Cov(r_p, r_f|E)}{\sqrt{Var(r_p|E)}\sqrt{Var(r_f|E)}}. \tag{A.3}$$

Using the formula for r_f in (A.2) and rewriting $Cov(r_p, r_f|E)$ leads to

$$Cov(r_p, r_f|E) = Cov(r_p, (\rho_{pf}\sigma_f/\sigma_p)r_p)|E) + Cov(r_p, \sqrt{1-\rho_{pf}^2}\sigma_f v|E). \tag{A.4}$$

By definition, the standard normal random variable v is independent of r_p. Thus, we eliminate the second term on the right hand side of (A.4) and obtain

$$Cov(r_p, r_f|E) = Cov(r_p, (\rho_{pf}\sigma_f/\sigma_p)r_p)|E) = (\rho_{pf}\sigma_f/\sigma_p)Var(r_p|E). \tag{A.5}$$

(A.2) can be rewritten as $r_f = \mu_f + (\rho_{pf}\sigma_f/\sigma_p)(r_p - \mu_p) + \sqrt{1-\rho_{pf}^2}\sigma_f v$. Using this formula for r_f and recalling that $Var(v) = 1$ we obtain

$$Var(r_f|E) = (\rho_{pf}^2\sigma_f^2/\sigma_p^2)Var(r_p|E) + (1-\rho_{pf}^2)\sigma_f^2. \tag{A.6}$$

Now, rewriting (A.3) using (A.5) and (A.6) leads to

$$\rho_E = \frac{(\rho_{pf}\sigma_f/\sigma_p)Var(r_p|E))}{\sqrt{Var(r_p|E)}\sqrt{(\rho_{pf}^2\sigma_f^2/\sigma_p^2)Var(r_p|E) + (1-\rho_{pf}^2)\sigma_f^2}}. \tag{A.7}$$

Simplifying this expression and using r_p for x and r_f for y, we obtain the formula given in (2) as

$$\rho_E = \rho_{pf}\Big(\rho_{pf}^2 + (1-\rho_{pf}^2)\frac{Var(r_p)}{Var(r_p|E)}\Big)^{-1/2}. \tag{A.8}$$

Next, we go back to the optimal hedge ratio formula in (1) and use it to define the conditional hedge ratio, h_E, as

$$h_E = -\rho_E\Big[\frac{Var(r_p|E)}{Var(r_f|E)}\Big]^{1/2}. \tag{A.9}$$

Using equation (A.6) for $Var(r_f|E)$ and the *unconditional* hedge ratio $h = \rho_{pf}(\sigma_p/\sigma_f)$ we obtain

$$\frac{h_E}{h} = \Big(\frac{\rho_E}{\rho_{pf}}\Big)\Big(\frac{Var(r_p|E)}{Var(r_p)}\Big)^{1/2}\Big(1-\rho_{pf}^2+\rho_{pf}^2\Big[\frac{Var(r_p|E)}{Var(r_p)}\Big]\Big)^{-1/2}.$$

Defining $k = Var(r_p|E)/Var(r_p)$ and using (A.8) for ρ_E, this equality simplifies to

$$\frac{h_E}{h} = \frac{k}{1-\rho_{pf}^2+k\rho_{pf}^2}.$$

Further simplifying this equality we obtain the expression in equation (3) stated in Theorem 1 as

$$h_E = hk\Big[1-\rho_{pf}^2+k\rho_{pf}^2\Big]^{-1}.$$

Proof of Corollary 1: Having formulated the conditional minimum-variance hedge ratio as in equation (3), we obtain that $h_E < h$ is satisfied when

$$k\Big[1-\rho_{pf}^2+k\rho_{pf}^2\Big]^{-1} = 1.$$

Solving the equality leads to the result in the corollary that $h_E < h$ is satisfied when $k < 1$. This means that if conditioning on the event leads to a smaller variance estimate for r_p, then the hedger will be mislead by the conditioning and buy h_E futures contracts even though the correct hedge ratio is h.

B The maximum likelihood procedure

The procedure is explained in detail for the multivariate case in Ledford and Tawn(1996), we apply the bivariate version to model spot and futures return dependencies. We can ignore the time subscript t for now as we explain the likelihood contribution of an observation for a given day. For a given marginal observation (r_p,r_f), the likelihood contribution $L(r_p, r_f)$ is given by

$$L(r_p, r_f) = \begin{cases} F(r_p, r_f) & \text{if } r_p < \theta_p, r_f < \theta_f \\ \frac{\partial F(r_p,r_f)}{\partial r_f} & \text{if } r_p < \theta_p, r_f > \theta_f \\ \frac{\partial F(r_p,r_f)}{\partial r_p} & \text{if } r_p > \theta_p, r_f < \theta_f \\ \frac{\partial^2 F(r_p,r_f)}{\partial r_p \partial r_f} & \text{if } r_p > \theta_p, r_f > \theta_f \end{cases}$$

The maximum likelihood procedure assumes that the only relevant information that is conveyed by a given pair of returns which do not exceed their thresholds is that the observation occurs below the threshold, not the actual values of returns. So, the procedure censors those observations that occur below the threshold.

Case 1 ($r_p < \theta_p$, $r_f < \theta_f$): None of the returns exceeds its associated threshold value. The likelihood contribution of this observation is given by

$$L(r_p, r_f) = \exp\Big[-D\Big(-\frac{1}{logF_p(\theta_p)}, -\frac{1}{logF_f(\theta_f)}\Big)\Big].$$

where the dependance function D is defined as in (8) and the univariate marginal distributions F_i ($i = f, p$) are defined as in (9). Note that the threshold values, not the actual observations, are used as inputs to the distribution functions

In order to simplify the proceeding formulas, define $t_i(r_i) = max\{0, 1+\xi_i \cdot (r_i - \theta_i)/\sigma_i\}^{-1/\xi_i}$ for ($i = f, p$). Then, the univariate marginal distribution for contract i can be expressed as $F_i = 1 - p_i t_i(r_i)$.

Case 2 ($r_p < \theta_p$, $r_f > \theta_f$): Only the return on the futures contract exceeds the threshold. The likelihood contribution is given by

$$L(r_p, r_f) = \frac{\partial F(r_p, r_f)}{\partial r_f} = \exp\Big[-D\Big(-\frac{1}{logF_p(\theta_p)}, -\frac{1}{logF_f(r_f)}\Big)\Big] \cdot \frac{\partial D}{\partial r_f}\Big|_{(\theta_p, r_f)} \cdot K_f$$

where $K_f = p_f \cdot (\sigma_f)^{-1} \cdot \{t_f(r_f)\}^{1+\xi_f} \cdot z_f^2 \cdot \exp(1/z_f)$ and $z_f = -1/logF_f(r_f)$.

Case 3 ($r_p > \theta_p$, $r_f < \theta_f$): Similar to Case 2, but in this case only the return on the spot contract exceeds the threshold. The likelihood contribution is given by

$$L(r_p, r_f) = \frac{\partial F(r_p, r_f)}{\partial r_p} = \exp\Big[-D\Big(-\frac{1}{logF_p(r_p)}, -\frac{1}{logF_f(\theta_f)}\Big)\Big] \cdot \frac{\partial D}{\partial r_p}\Big|_{(r_p, \theta_f)} \cdot K_p$$

where $K_p = p_p \cdot (\sigma_p)^{-1} \cdot \{t_p(r_p)\}^{1+\xi_p} \cdot z_p^2 \cdot \exp(1/z_p)$ and $z_p = -1/logF_p(r_p)$.

Case 4 ($r_p > \theta_p$, $r_f > \theta_f$): Both returns exceed their corresponding thresholds, the likelihood contribution is given by

$$L(r_p, r_f) = \frac{\partial^2 F(r_p, r_f)}{\partial r_p \partial r_f} = \exp\Big[-D\Big(-\frac{1}{logF_p(r_p)}, -\frac{1}{logF_f(r_f)}\Big)\Big] \cdot \Big[\frac{\partial D}{\partial r_p} \cdot \frac{\partial D}{\partial r_f} - \frac{\partial^2 D}{\partial r_p \partial r_f}\Big]_{(r_p, r_f)} \cdot K_p \cdot K_f$$

where K_p and K_f are formulated as given in cases 2 and 3.

C Several Properties of the Minimum LPM Hedge Ratio

This appendix presents several results on how optimal hedge ratios, in theory, should behave with respect to target rates (c) and orders of moment (n) chosen. Consider a long hedger whose goal is to minimize exposure to the region in the distribution for the return on the hedge portfolio, r_l, such that $r_l = -r_p + hr_f \leq c$ where h ($h \geq 0$) is the hedge ratio. The general expression for the lower partial moment to be minimized is

$$L(c, n, r_s) = E\{[\max(0, c + r_p - hr_f)]^n\}. \tag{C.1}$$

The first order condition is obtained by taking the derivative of (C.1) with respect to h,

$$nE\{[\max(0, c + r_p - h^* r_f)]^{n-1} r_f\} = 0. \tag{C.2}$$

One can see that the first order condition may be satisfied at multiple optimal h^* values if a sufficiently negative c value is chosen, i.e. global optimality is not guaranteed for those cases where the maximum term takes on the value of zero for all possible (r_p, r_f) pairs. The second order condition, however, should be analyzed separately for different values of n. For simplicity denote $L(c, n, r_s)$ function of equation (C.1) as L_n and let $g(r_p, r_f)$ be the joint probability distribution of the spot and futures returns.
Case 1 ($n = 1$): We can rewrite the LPM function as

$$L_1(c, h) = \int_{-\infty}^{\infty} \int_{-\infty}^{c+hr_f} (c - r_p + hr_f) g(r_p, r_f) dr_p dr_f \tag{C.3}$$

with

$$\begin{aligned} \frac{\partial L_1}{\partial h} &= \int_{-\infty}^{\infty} \int_{-\infty}^{c+hr_f} r_f g(r_p, r_f) dr_p dr_f \\ \frac{\partial^2 L_1}{\partial h^2} &= \int_{-\infty}^{\infty} r_f^2 g(c + hr_f, r_f) dr_f. \end{aligned}$$

At the optimal hedge ratio, $h = h^*$, taking the derivative of the first order condition with respect to c leads to

$$\left(\frac{\partial^2 L_1}{\partial h \partial c} + \frac{\partial^2 L_1}{\partial h^2} \frac{\partial h}{\partial c} \right)_{h=h^*} = 0 \tag{C.4}$$

where

$$\frac{\partial^2 L_1}{\partial h \partial c} = \int_{-\infty}^{\infty} r_f g(c + hr_f, r_f) dr_f.$$

Thus we obtain,

$$\frac{\partial h^*}{\partial c} = -\frac{\int_{-\infty}^{\infty} r_f g(c + hr_f, r_f) dr_f}{\int_{-\infty}^{\infty} r_f^2 g(c + hr_f, r_f) dr_f}. \tag{C.5}$$

Now, define a marginal probability distribution $g^{'}(r_f) = g(c + hr_f, r_f)$. Then, (C.5) can be rewritten as

$$\frac{\partial h^*}{\partial c} = -\frac{E_{g'}(r_f)}{E_{g'}(r_f^2)}. \tag{C.6}$$

where $E_{g'}(.)$ denotes expectation under $g^{'}$. Examining (C.6), one can see that even though the denominator is always nonnegative, the numerator's sign is ambiguous. Note the expectation is defined under the marginal distribution $g^{'}$, therefore the sign of the numerator depends on the marginal distribution. Thus, the behavior of the optimal hedge ratio h^* with respect to c is undetermined.

Case 2 ($n = 2$): We can rewrite the LPM function as

$$L_2(c, h) = \int_{-\infty}^{\infty} \int_{-\infty}^{c+hr_f} (c - r_p + hr_f)^2 g(r_p, r_f) dr_p dr_f. \tag{C.7}$$

Similar to Case 1, the partial derivatives can be rewritten as

$$\begin{aligned}
\frac{\partial L_2}{\partial h} &= 2\int_{-\infty}^{\infty} \int_{-\infty}^{c+hr_f} r_f(c - r_p + hr_f) g(r_p, r_f) dr_p dr_f \\
\frac{\partial^2 L_2}{\partial h^2} &= 2\int_{-\infty}^{\infty} \int_{-\infty}^{c+hr_f} r_f^2 g(r_p, r_f) dr_p dr_f \\
\frac{\partial^2 L_2}{\partial h \partial c} &= 2\int_{-\infty}^{\infty} \int_{-\infty}^{c+hr_f} r_f g(r_p, r_f) dr_p dr_f.
\end{aligned}$$

Regarding the partial derivative of the optimal hedge ratio with respect to the target rate, we obtain

$$\frac{\partial h^*}{\partial c} = -\frac{\int_{-\infty}^{\infty} \int_{-\infty}^{c+hr_f} r_f g(r_p, r_f) dr_p dr_f}{\int_{-\infty}^{\infty} \int_{-\infty}^{c+hr_f} r_f^2 g(r_p, r_f) dr_p dr_f}. \tag{C.8}$$

Once again examining (C.8), one can see that the denominator is always nonnegative. The term in the numerator can be rewritten as $E[r_f \cdot I_{r_p - hr_f \leq c}]$ where $I_{r_p - hr_f \leq c} = 1$ if $r_p - hr_f \leq c$; 0 otherwise. The numerator then can be rewritten as

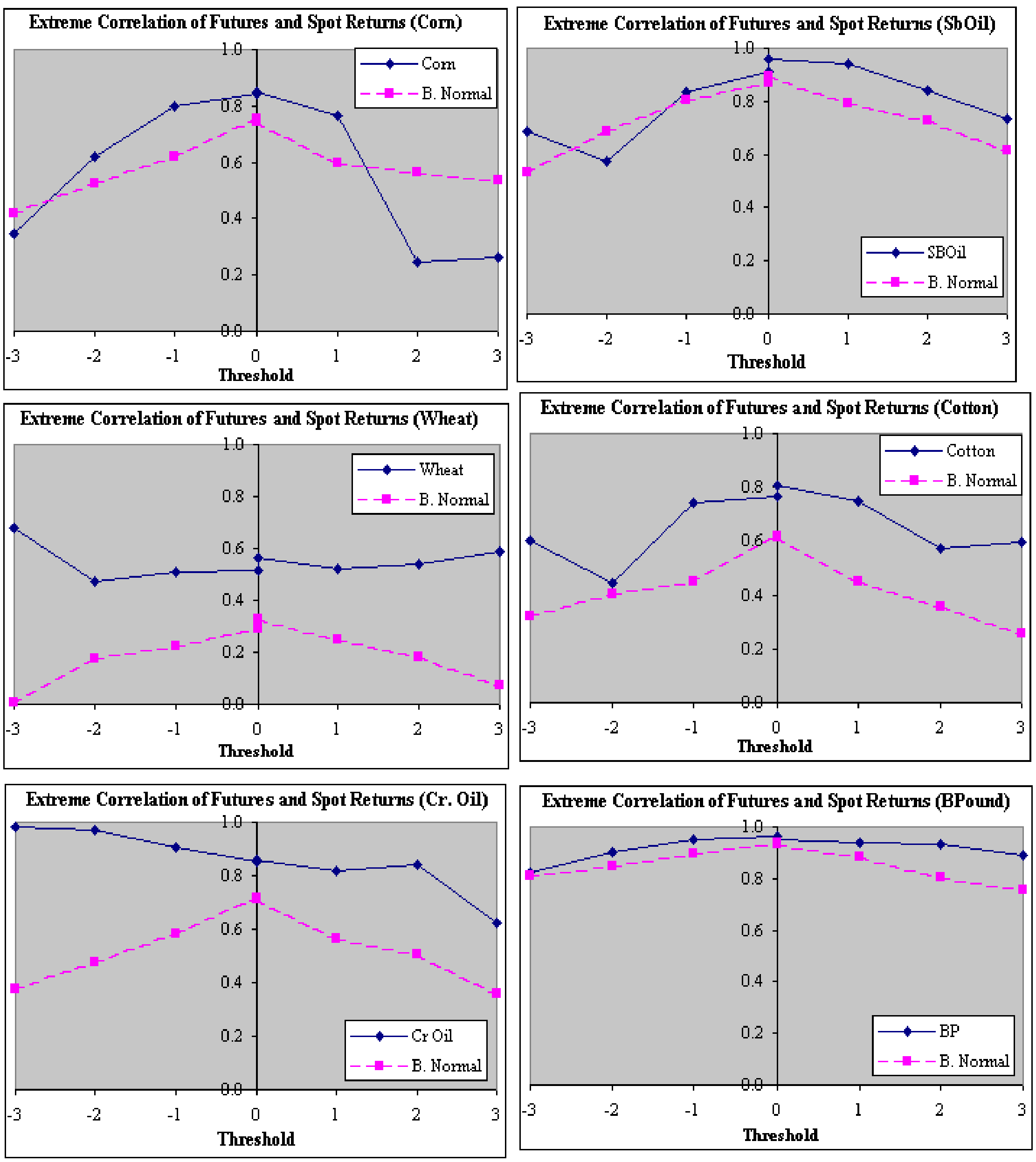
$$E[r_f \cdot I_{r_p - hr_f \leq c}] = Cov(r_f, I_{r_p - hr_f \leq c}) + E[I_{r_p - hr_f \leq c}] \cdot E[r_f]$$

In order to determine the sign of the covariance term between the indicator function and r_f, one needs to specify the conditional distribution function for (r_p, r_f) in the region where $r_p - hr_f \leq c$. Since higher values of r_f will usually be associated with higher values of r_p, the sign of the covariance term will be ambiguous unless the conditional covariance and variance terms, $Cov(r_p, r_f | r_p - hr_f \leq c)$ and $Var(r_f | r_p - hr_f \leq c)$, are specified. Similarly, the second term $E[r_f]$ can take on any sign. Therefore, the sign of the partial derivative in (C.8) is undetermined once again.

Table 1: Descriptive statistics of futures and spot returns.
(ρ_{pf} refers to correlation between spot and futures returns and $r(i)$ refers to the i-th order autocorrelation coefficient)

Return Series	**Soybean Oil**		**Corn**		**Cotton**		**Crude Oil**		**Wheat**	
	Spot	Futures	Spot	Futures	Spot	Futures	Spot	Futures	Spot	Futures
n	2659		2658		2638		2631		2658	
Mean (10^{-4})	0.730	-1.030	0.803	-0.679	0.906	0.777	-1.070	2.466	-0.385	2.039
Std. Dev. (10^{-2})	1.388	1.322	1.478	1.291	1.417	1.268	2.488	2.252	1.289	1.246
Skewness	0.009	0.105	-0.553	-0.076	-0.133	-0.086	-1.706	-2.138	-0.041	0.209
Kurtosis	2.267	1.585	6.699	3.492	1.384	0.438	32.126	43.190	4.513	2.679
Min. (10^{-2})	-8.257	-5.778	-11.937	-8.822	-7.199	-5.085	-40.204	-40.048	-9.689	-6.210
Max. (10^{-2})	6.398	6.338	8.923	6.968	8.068	4.182	18.297	13.572	7.460	6.732
ρ_{pf}	0.951		0.891		0.820		0.874		0.565	
r(1)	0.056	0.083	0.036	0.113	0.040	0.052	-0.007	-0.002	0.053	0.092
r(2)	-0.041	-0.038	0.000	0.017	-0.054	-0.062	-0.043	-0.027	-0.022	-0.042
r(3)	-0.013	-0.026	-0.019	-0.022	-0.023	-0.017	-0.077	-0.091	0.023	-0.011
r(4)	0.017	0.016	0.020	-0.007	0.020	0.031	-0.027	-0.028	0.020	0.025
Return Series	**Japanese Yen**		**British Pound**		**Deutsche Mark**		**S&P 500**		**NYSE 100**	
	Spot	Futures	Spot	Futures	Spot	Futures	Spot	Futures	Spot	Futures
n	2640		2640		2640		2644		2645	
Mean (10^{-4})	-0.652	-1.665	-0.517	0.633	-0.492	-0.408	5.536	4.318	5.190	4.122
Std. Dev. (10^{-2})	0.701	0.710	0.666	0.685	0.691	0.698	0.826	0.903	0.747	0.883
Skewness	0.304	0.261	-0.209	-0.193	-0.015	-0.009	-0.663	-0.975	-0.816	-1.189
Kurtosis	3.639	4.114	3.380	3.393	2.125	2.271	7.113	10.957	7.841	14.151
Min. (10^{-2})	-3.963	-4.207	-4.331	-4.476	-3.484	-3.312	-7.113	-8.782	-6.793	-9.486
Max. (10^{-2})	4.679	4.753	3.247	3.475	3.344	3.601	4.989	5.617	4.109	5.811
ρ_{pf}	0.970		0.976		0.973		0.966		0.957	
r(1)	0.000	-0.005	0.030	0.018	0.009	0.026	0.015	-0.053	0.059	-0.071
r(2)	-0.023	-0.019	-0.005	-0.004	-0.019	-0.031	-0.028	-0.047	-0.020	-0.040
r(3)	-0.029	-0.035	-0.018	-0.009	-0.015	-0.017	-0.044	-0.018	-0.046	-0.016
r(4)	0.015	0.011	0.031	0.015	-0.004	-0.005	-0.024	-0.027	-0.025	-0.023

Figure 1: Exceedance correlations of futures and spot returns at selected thresholds.



(Figure 1 cont'd)

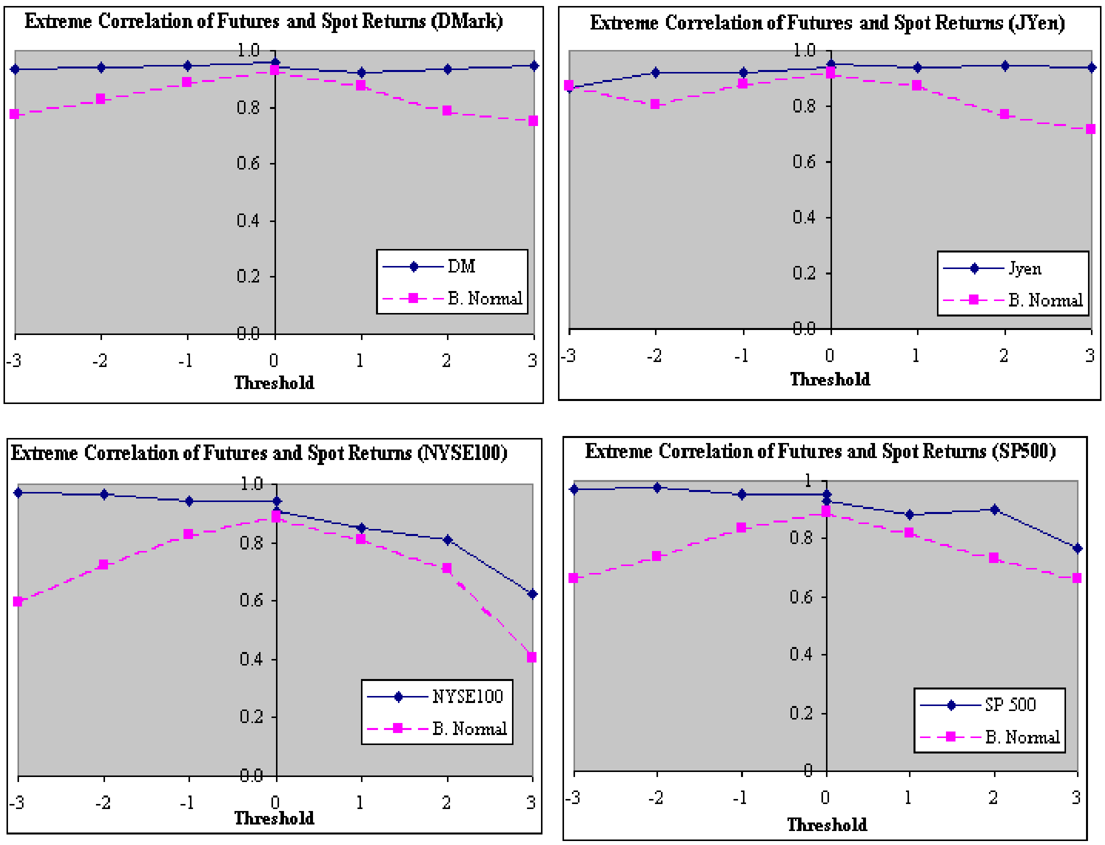


Table 2: Parameters of the bivariate distribution of spot and futures return exceedances. (Threshold values are defined in terms of standard deviations away from the empirical mean of spot and futures returns for the corresponding commodity. Standard deviations are given in parentheses)

Soybean Oil							
	Model Parameters, Φ						
Threshold	**Spot**			**Futures**			**Correlation**
θ	p_p	σ_p	ξ_p	p_f	σ_f	ξ_f	ρ^e_{pf}
	Panel A: Negative Return Exceedances						
-3	0.003	0.009	-0.243	0.004	0.003	-0.189	0.685
	(0.017)	(0.044)	(7.302)	(0.003)	(0.003)	(1.545)	
-2	0.019	0.005	0.105	0.017	0.005	0.174	0.572
	(0.003)	(0.001)	(0.382)	(0.003)	(0.001)	(0.352)	
-1	0.101	0.006	-0.096	0.132	0.009	0.206	0.833
	(0.002)	(0.000)	(0.042)	(0.002)	(0.000)	(0.035)	
0	0.499	0.008	0.206	0.498	0.007	0.208	0.910
	(0.000)	(0.000)	(0.003)	(0.001)	(0.000)	(0.003)	
	Panel B: Positive Return Exceedances						
0	0.495	0.009	0.212	0.494	0.009	0.214	0.957
	(0.001)	(0.000)	(0.005)	(0.001)	(0.000)	(0.005)	
+1	0.128	0.009	0.221	0.127	0.009	0.224	0.938
	(0.002)	(0.000)	(0.035)	(0.002)	(0.000)	(0.036)	
+2	0.025	0.007	-0.250	0.032	0.006	-0.304	0.838
	(0.003)	(0.001)	(0.265)	(0.002)	(0.000)	(0.151)	
+3	0.007	0.006	-0.338	0.005	0.005	-0.407	0.735
	(0.005)	(0.004)	(1.211)	(0.002)	(0.003)	(0.936)	

(Table 2 cont'd)

Corn							
	Model Parameters, Φ						
Threshold	**Spot**			**Futures**			**Correlation**
θ	p_p	σ_p	ξ_p	p_f	σ_f	ξ_f	ρ^e_{pf}
Panel A: Negative Return Exceedances							
-3	0.010	0.007	0.208	0.009	0.007	0.237	0.347
	(0.006)	(0.005)	(1.164)	(0.008)	(0.005)	(1.662)	
-2	0.029	0.007	-0.240	0.026	0.018	-0.292	0.620
	(0.004)	(0.001)	(0.222)	(0.017)	(0.001)	(1.008)	
-1	0.117	0.009	-0.301	0.096	0.013	-0.336	0.798
	(0.002)	(0.000)	(0.037)	(0.003)	(0.000)	(0.055)	
0	0.486	0.009	-0.338	0.519	0.119	-0.339	0.845
	(0.001)	(0.000)	(0.003)	(0.001)	(0.000)	(0.004)	
Panel B: Positive Return Exceedances							
0	0.509	0.009	-0.339	0.469	0.008	-0.339	0.846
	(0.000)	(0.000)	(0.003)	(0.000)	(0.000)	(0.003)	
+1	0.119	0.001	-0.001	0.109	0.009	-0.017	0.766
	(0.001)	(0.000)	(0.014)	(0.000)	(0.004)	(0.009)	
+2	0.023	0.008	0.151	0.019	0.009	0.171	0.247
	(0.003)	(0.001)	(0.219)	(0.003)	(0.001)	(0.290)	
+3	0.009	0.005	-0.191	0.007	0.011	-0.249	0.263
	(0.006)	(0.003)	(1.372)	(0.013)	(0.020)	(2.820)	
Cotton							
Panel A: Negative Return Exceedances							
-3	0.011	0.005	-0.190	0.004	0.006	-0.463	0.601
	(0.007)	(0.003)	(0.906)	(0.008)	(0.012)	(2.519)	
-2	0.032	0.004	0.468	0.036	0.008	0.472	0.446
	(0.001)	(0.000)	(0.065)	(0.005)	(0.001)	(0.192)	
-1	0.134	0.007	-0.472	0.131	0.006	-0.473	0.741
	(0.000)	(0.000)	(0.009)	(0.008)	(0.000)	(0.009)	
0	0.509	0.008	-0.473	0.519	0.009	-0.474	0.764
	(0.000)	(0.000)	(0.002)	(0.000)	(0.000)	(0.003)	
Panel B: Positive Return Exceedances							
0	0.487	0.006	-0.474	0.468	0.009	-0.474	0.809
	(0.005)	(0.000)	(0.001)	(0.000)	(0.000)	(0.002)	
+1	0.149	0.007	0.473	0.146	0.005	0.475	0.746
	(0.001)	(0.000)	(0.008)	(0.006)	(0.000)	(0.006)	
+2	0.026	0.002	0.475	0.020	0.006	0.482	0.575
	(0.009)	(0.001)	(0.053)	(0.002)	(0.001)	(0.158)	
+3	0.003	0.002	-0.054	0.002	0.002	-0.366	0.595
	(0.049)	(0.029)	(20.684)	(0.005)	(0.004)	(3.499)	

(Table 2 cont'd)

Crude Oil							
	Model Parameters, Φ						
Threshold	**Spot**			**Futures**			**Correlation**
θ	p_p	σ_p	ξ_p	p_f	σ_f	ξ_f	ρ^e_{pf}
Panel A: Negative Return Exceedances							
-3	0.004	0.060	0.182	0.005	0.053	-0.161	0.981
	(0.121)	(1.388)	(21.92)	(0.109)	(1.128)	(16.506)	
-2	0.011	0.039	0.319	0.016	0.047	0.367	0.973
	(0.026)	(0.100)	(3.755)	(0.039)	(0.115)	(3.269)	
-1	0.086	0.029	-0.375	0.073	0.026	-0.402	0.906
	(0.009)	(0.003)	(0.156)	(0.008)	(0.003)	(0.150)	
0	0.495	0.019	0.419	0.496	0.019	0.419	0.850
	(0.002)	(0.000)	(0.006)	(0.001)	(0.000)	(0.005)	
Panel B: Positive Return Exceedances							
0	0.497	0.015	0.420	0.509	0.016	0.420	0.856
	(0.001)	(0.000)	(0.004)	(0.002)	(0.000)	(0.005)	
+1	0.089	0.018	-0.425	0.088	0.021	-0.427	0.815
	(0.004)	(0.000)	(0.072)	(0.005)	(0.001)	(0.086)	
+2	0.021	0.019	0.439	0.0169	0.027	0.458	0.838
	(0.008)	(0.009)	(0.599)	(0.001)	(0.019)	(1.041)	
+3	0.004	0.015	0.474	0.006	0.027	0.476	0.621
	(0.009)	(0.044)	(2.324)	(0.015)	(0.092)	(2.858)	
Wheat							
Panel A: Negative Return Exceedances							
-3	0.004	0.006	0.102	0.004	0.008	0.335	0.679
	(0.007)	(0.009)	(3.162)	(0.007)	(0.015)	(3.429)	
-2	0.028	0.006	0.094	0.030	0.009	0.241	0.471
	(0.004)	(0.001)	(0.316)	(0.005)	(0.001)	(0.320)	
-1	0.119	0.008	0.244	0.119	0.009	0.245	0.512
	(0.002)	(0.000)	(0.033)	(0.003)	(0.000)	(0.042)	
0	0.497	0.007	-0.248	0.365	0.008	-0.137	0.516
	(0.000)	(0.000)	(0.003)	(0.001)	(0.000)	(0.007)	
Panel B: Positive Return Exceedances							
0	0.381	0.007	-0.098	0.376	0.007	-0.108	0.563
	(0.001)	(0.000)	(0.005)	(0.001)	(0.000)	(0.006)	
+1	0.118	0.007	-0.116	0.099	0.009	-0.170	0.519
	(0.002)	(0.000)	(0.038)	(0.003)	(0.000)	(0.054)	
+2	0.025	0.009	-0.187	0.024	0.008	-0.211	0.538
	(0.006)	(0.002)	(0.468)	(0.005)	(0.001)	(0.423)	
+3	0.007	0.008	0.239	0.007	0.008	0.277	0.589
	(0.008)	(0.010)	(1.864)	(0.007)	(0.011)	(1.941)	

(Table 2 cont'd)

Japanese Yen							
	Model Parameters, Φ						
Threshold	**Spot**			**Futures**			**Correlation**
θ	p_p	σ_p	ξ_p	p_f	σ_f	ξ_f	ρ^e_{pf}
Panel A: Negative Return Exceedances							
-3	0.003	0.004	-0.248	0.002	0.005	-0.279	0.863
	(0.008)	(0.013)	(4.031)	(0.006)	(0.015)	(3.628)	
-2	0.019	0.005	-0.312	0.021	0.005	-0.320	0.921
	(0.003)	(0.001)	(0.311)	(0.003)	(0.000)	(0.281)	
-1	0.117	0.004	-0.329	0.130	0.005	-0.326	0.922
	(0.001)	(0.000)	(0.017)	(0.002)	(0.000)	(0.016)	
0	0.509	0.003	-0.327	0.512	0.004	-0.328	0.941
	(0.000)	(0.000)	(0.001)	(0.000)	(0.000)	(0.002)	
Panel B: Positive Return Exceedances							
0	0.498	0.004	-0.328	0.495	0.003	-0.330	0.948
	(0.000)	(0.000)	(0.002)	(0.000)	(0.000)	(0.001)	
+1	0.119	0.005	-0.331	0.116	0.005	-0.339	0.937
	(0.001)	(0.000)	(0.002)	(0.001)	(0.000)	(0.017)	
+2	0.028	0.005	-0.393	0.027	0.005	-0.398	0.945
	(0.002)	(0.005)	(0.112)	(0.002)	(0.006)	(0.126)	
+3	0.006	0.006	-0.429	0.006	0.006	-0.432	0.937
	(0.003)	(0.004)	(0.776)	(0.003)	(0.004)	(0.784)	
British Pound							
Panel A: Negative Return Exceedances							
-3	0.006	0.003	-0.165	0.005	0.003	-0.276	0.822
	(0.004)	(0.002)	(1.289)	(0.003)	(0.001)	(1.018)	
-2	0.033	0.003	-0.277	0.032	0.003	-0.278	0.905
	(0.001)	(0.000)	(0.079)	(0.001)	(0.000)	(0.071)	
-1	0.115	0.005	-0.283	0.113	0.005	-0.290	0.948
	(0.001)	(0.000)	(0.002)	(0.001)	(0.000)	(0.018)	
0	0.488	0.005	-0.291	0.508	0.004	-0.292	0.961
	(0.001)	(0.000)	(0.004)	(0.000)	(0.000)	(0.002)	
Panel B: Positive Return Exceedances							
0	0.509	0.003	-0.293	0.509	0.005	-0.293	0.953
	(0.000)	(0.000)	(0.002)	(0.000)	(0.000)	(0.001)	
+1	0.118	0.004	-0.294	0.117	0.004	-0.296	0.939
	(0.001)	(0.000)	(0.016)	(0.001)	(0.000)	(0.017)	
+2	0.022	0.005	-0.321	0.023	0.003	-0.337	0.932
	(0.002)	(0.000)	(0.198)	(0.001)	(0.001)	(0.129)	
+3	0.007	0.004	-0.351	0.008	0.005	-0.352	0.888
	(0.003)	(0.002)	(0.765)	(0.003)	(0.002)	(0.744)	

(Table 2 cont'd)

Deutsche Mark							
	Model Parameters, Φ						
Threshold		**Spot**			**Futures**		**Correlation**
θ	p_p	σ_p	ξ_p	p_f	σ_f	ξ_f	ρ^e_{pf}
	Panel A: Negative Return Exceedances						
-3	0.005	0.003	-0.228	0.004	0.003	-0.257	0.932
	(0.004)	(0.003)	(1.762)	(0.003)	(0.002)	(1.301)	
-2	0.027	0.005	-0.259	0.027	0.004	-0.261	0.937
	(0.003)	(0.000)	(0.189)	(0.003)	(0.000)	(0.189)	
-1	0.123	0.004	-0.275	0.126	0.005	-0.286	0.945
	(0.001)	(0.000)	(0.016)	(0.001)	(0.000)	(0.014)	
0	0.486	0.004	-0.288	0.505	0.004	-0.292	0.956
	(0.000)	(0.000)	(0.001)	(0.000)	(0.000)	(0.000)	
	Panel B: Positive Return Exceedances						
0	0.498	0.004	-0.297	0.498	0.003	-0.296	0.942
	(0.000)	(0.000)	(0.002)	(0.000)	(0.000)	(0.001)	
+1	0.126	0.004	-0.302	0.128	0.004	-0.309	0.925
	(0.001)	(0.000)	(0.014)	(0.001)	(0.000)	(0.013)	
+2	0.023	0.005	-0.328	0.024	0.004	-0.328	0.936
	(0.002)	(0.004)	(0.138)	(0.002)	(0.000)	(0.139)	
+3	0.004	0.003	-0.396	0.007	0.002	-0.402	0.945
	(0.002)	(0.010)	(0.827)	(0.002)	(0.001)	(0.608)	
S&P 500							
	Panel A: Negative Return Exceedances						
-3	0.005	0.011	-0.250	0.005	0.013	-0.214	0.973
	(0.022)	(0.052)	(6.720)	(0.029)	(0.063)	(7.269)	
-2	0.026	0.007	-0.122	0.027	0.006	-0.098	0.974
	(0.006)	(0.001)	(0.416)	(0.005)	(0.001)	(0.361)	
-1	0.101	0.006	-0.093	0.104	0.007	-0.076	0.953
	(0.002)	(0.000)	(0.043)	(0.002)	(0.000)	(0.043)	
0	0.413	0.0003	-0.052	0.422	0.0004	-0.043	0.951
	(0.000)	(0.000)	(0.004)	(0.000)	(0.000)	(0.003)	
	Panel B: Positive Return Exceedances						
0	0.398	0.0003	-0.073	0.406	0.0003	-0.066	0.932
	(0.000)	(0.000)	(0.002)	(0.000)	(0.000)	(0.002)	
+1	0.129	0.003	-0.086	0.133	0.004	-0.078	0.884
	(0.001)	(0.000)	(0.019)	(0.001)	(0.000)	(0.019)	
+2	0.019	0.004	-0.130	0.021	0.004	-0.118	0.901
	(0.003)	(0.000)	(0.297)	(0.003)	(0.001)	(0.324)	
+3	0.005	0.004	-0.192	0.005	0.006	-0.213	0.767
	(0.007)	(0.010)	(2.468)	(0.008)	(0.011)	(3.214)	

(Table 2 cont'd)

NYSE Composite							
	Model Parameters, Φ						
Threshold	**Spot**			**Futures**			**Correlation**
θ	p_p	σ_p	ξ_p	p_f	σ_f	ξ_f	ρ^e_{pf}
Panel A: Negative Return Exceedances							
-3	0.004	0.018	-0.378	0.005	0.013	-0.474	0.968
	(0.018)	(0.136)	(7.253)	(0.016)	(0.042)	(2.420)	
-2	0.025	0.010	-0.463	0.031	0.009	-0.464	0.966
	(0.003)	(0.001)	(0.147)	(0.003)	(0.001)	(0.170)	
-1	0.091	0.008	-0.480	0.117	0.007	-0.481	0.945
	(0.001)	(0.000)	(0.022)	(0.001)	(0.000)	(0.013)	
0	0.476	0.0006	-0.482	0.475	0.005	-0.482	0.944
	(0.000)	(0.000)	(1.704)	(0.000)	(0.000)	(0.001)	
Panel B: Positive Return Exceedances							
0	0.506	0.005	-0.482	0.506	0.004	-0.482	0.909
	(0.000)	(0.000)	(0.001)	(0.000)	(0.000)	(0.000)	
+1	0.102	0.006	-0.484	0.114	0.004	-0.486	0.848
	(0.001)	(0.000)	(0.012)	(0.000)	(0.000)	(0.008)	
+2	0.016	0.007	-0.489	0.017	0.004	-0.492	0.811
	(0.002)	(0.001)	(0.186)	(0.001)	(0.000)	(0.101)	
+3	0.004	0.009	-0.494	0.002	0.003	-0.497	0.626
	(0.008)	(0.019)	(1.341)	(0.003)	(0.013)	(2.056)	

Table 3: Estimated optimal hedge ratios and hedging performances for given (c,n) pairs. (h^* denotes the *minimum-variance* hedge ratio. In the first column, target return (c) denotes the number of standard deviations away from the empirical mean of spot returns for the corresponding commodity. For a given target return, the first line refers to the short hedger and the second line refers to the long hedger. In each cell, the first number is the optimal hedge ratio and the second number, i.e. the number in parenthesis, is the hedging performance)

	Soybean Oil ($h^* = 0.997$)		**Corn** ($h^* = 1.019$)	
c	n=1	n=2	n=1	n=2
-3	1.116 (0.887)	0.998 (0.771)	1.148 (0.908)	1.248 (0.801)
	0.395 (0.970)	0.351 (0.932)	0.893 (0.885)	0.902 (0.803)
-2	1.446 (0.884)	1.176 (0.657)	1.230 (0.847)	1.212 (0.702)
	0.720 (0.935)	0.520 (0.727)	0.939 (0.855)	0.953 (0.668)
-1.5	1.301 (0.897)	1.262 (0.659)	1.139 (0.820)	1.201 (0.658)
	0.744 (0.941)	0.628 (0.730)	0.920 (0.848)	0.930 (0.643)
-1	1.144 (0.914)	1.232 (0.676)	1.058 (0.813)	1.158 (0.622)
	0.886 (0.933)	0.732 (0.731)	0.964 (0.857)	0.931 (0.629)
-0.5	1.056 (0.924)	1.138 (0.696)	1.022 (0.797)	1.102 (0.594)
	0.963 (0.920)	0.850 (0.722)	0.998 (0.851)	0.974 (0.626)
0	1.007 (0.850)	1.016 (0.693)	1.017 (0.623)	1.038 (0.537)
	1.006 (0.829)	0.979 (0.687)	1.011 (0.609)	0.998 (0.556)
+0.5	1.023 (0.255)	1.013 (0.438)	1.029 (0.210)	1.028 (0.361)
	0.899 (0.230)	0.987 (0.430)	0.997 (0.193)	1.000 (0.343)
+1	1.151 (0.079)	1.015 (0.246)	1.135 (0.061)	1.030 (0.209)
	0.769 (0.059)	0.985 (0.237)	0.985 (0.059)	0.999 (0.193)

(Table 3 cont'd)

	Cotton ($h^* = 0.915$)		**Crude Oil** ($h^* = 0.966$)	
c	n=1	n=2	n=1	n=2
-3	0.809 (0.432)	0.943 (0.219)	1.063 (0.775)	1.308 (0.757)
	0.374 (0.224)	0.605 (0.178)	1.026 (0.762)	0.907 (0.434)
-2	0.572 (0.564)	0.749 (0.266)	0.935 (0.692)	1.136 (0.633)
	0.477 (0.582)	0.478 (0.234)	0.830 (0.806)	0.878 (0.492)
-1.5	0.744 (0.666)	0.708 (0.329)	0.928 (0.687)	1.032 (0.583)
	0.636 (0.674)	0.535 (0.333)	0.808 (0.807)	0.869 (0.516)
-1	0.681 (0.705)	0.674 (0.394)	0.941 (0.708)	0.978 (0.545)
	0.686 (0.721)	0.623 (0.408)	0.867 (0.805)	0.860 (0.532)
-0.5	0.843 (0.700)	0.743 (0.425)	0.972 (0.740)	0.963 (0.521)
	0.837 (0.716)	0.731 (0.441)	0.922 (0.807)	0.882 (0.545)
0	1.035 (0.649)	0.916 (0.420)	0.996 (0.662)	0.976 (0.501)
	1.037 (0.664)	0.914 (0.433)	0.999 (0.693)	0.954 (0.532)
+0.5	0.815 (0.187)	0.940 (0.296)	0.871 (0.165)	0.969 (0.339)
	0.864 (0.197)	0.949 (0.303)	1.017 (0.187)	0.974 (0.340)
+1	0.642 (0.052)	0.926 (0.174)	0.769 (0.041)	0.958 (0.191)
	0.705 (0.062)	0.938 (0.178)	1.144 (0.061)	0.978 (0.188)
	Wheat ($h^* = 0.584$)		**Japanese Yen** ($h^* = 0.956$)	
c	n=1	n=2	n=1	n=2
-3	0.554 (0.415)	0.741 (0.340)	0.481 (1.000)	0.481 (1.000)
	0.581 (0.619)	0.648 (0.412)	0.544 (1.000)	0.544 (1.000)
-2	0.696 (0.428)	0.686 (0.276)	0.824 (1.000)	0.824 (1.000)
	0.585 (0.572)	0.621 (0.375)	0.695 (1.000)	0.695 (1.000)
-1.5	0.626 (0.401)	0.669 (0.258)	0.996 (0.995)	1.036 (0.963)
	0.641 (0.530)	0.612 (0.354)	0.874 (0.998)	0.889 (0.989)
-1	0.530 (0.356)	0.612 (0.234)	0.874 (0.986)	0.950 (0.925)
	0.629 (0.435)	0.627 (0.318)	0.951 (0.988)	0.935 (0.942)
-0.5	0.526 (0.286)	0.570 (0.204)	0.932 (0.970)	0.926 (0.881)
	0.610 (0.320)	0.621 (0.261)	0.949 (0.964)	0.939 (0.888)
0	0.554 (0.158)	0.558 (0.158)	0.970 (0.800)	0.956 (0.764)
	0.592 (0.184)	0.611 (0.193)	0.957 (0.737)	0.956 (0.749)
+0.5	0.543 (0.067)	0.558 (0.104)	1.023 (0.290)	0.969 (0.461)
	0.579 (0.085)	0.602 (0.126)	0.835 (0.210)	0.945 (0.436)
+1	0.574 (0.024)	0.556 (0.064)	1.211 (0.107)	0.980 (0.261)
	0.597 (0.034)	0.599 (0.079)	0.644 (0.052)	0.933 (0.237)

(Table 3 cont'd)

	British Pound ($h^* = 0.948$)		**Deutsche Mark** ($h^* = 0.962$)	
c	n=1	n=2	n=1	n=2
-3	0.594 (1.000)	0.594 (1.000)	0.433 (1.000)	0.433 (1.000)
	0.375 (1.000)	0.375 (1.000)	0.355 (1.000)	0.355 (1.000)
-2	0.900 (1.000)	0.900 (1.000)	0.643 (1.000)	0.643 (1.000)
	0.598 (1.000)	0.598 (1.000)	0.812 (1.000)	0.812 (1.000)
-1.5	1.052 (0.999)	1.043 (0.993)	0.750 (1.000)	0.750 (1.000)
	0.799 (1.000)	0.799 (1.000)	1.120 (1.000)	1.120 (1.000)
-1	0.947 (0.994)	1.017 (0.953)	0.858 (0.993)	0.889 (0.964)
	1.024 (0.996)	0.991 (0.978)	1.002 (0.992)	1.085 (0.952)
-0.5	0.942 (0.978)	0.946 (0.914)	0.940 (0.976)	0.918 (0.908)
	0.930 (0.985)	0.937 (0.933)	0.958 (0.977)	0.997 (0.903)
0	0.953 (0.766)	0.949 (0.774)	0.969 (0.787)	0.957 (0.770)
	0.957 (0.791)	0.947 (0.781)	0.968 (0.771)	0.967 (0.765)
+0.5	0.890 (0.235)	0.944 (0.452)	0.992 (0.267)	0.964 (0.460)
	0.976 (0.267)	0.953 (0.460)	0.899 (0.250)	0.960 (0.449)
+1	0.744 (0.060)	0.939 (0.248)	1.146 (0.082)	0.967 (0.257)
	1.165 (0.089)	0.957 (0.256)	0.791 (0.072)	0.956 (0.249)

	S&P 500 ($h^* = 0.882$)		**NYSE Composite** ($h^* = 0.809$)	
c	n=1	n=2	n=1	n=2
-3	0.609 (1.000)	0.609 (1.000)	0.549 (1.000)	0.549 (1.000)
	0.457 (1.000)	0.457 (1.000)	0.313 (1.000)	0.313 (1.000)
-2	0.716 (1.000)	0.717 (1.000)	0.638 (1.000)	0.638 (1.000)
	0.604 (1.000)	0.604 (1.000)	0.468 (1.000)	0.468 (1.000)
-1.5	0.770 (1.000)	0.770 (1.000)	0.682 (0.996)	0.667 (0.980)
	0.798 (1.000)	0.798 (1.000)	0.635 (1.000)	0.635 (1.000)
-1	0.823 (0.995)	0.801 (0.972)	0.735 (0.982)	0.696 (0.931)
	0.811 (0.995)	0.824 (0.968)	0.715 (0.995)	0.724 (0.967)
-0.5	0.876 (0.959)	0.837 (0.904)	0.804 (0.930)	0.760 (0.855)
	0.866 (0.966)	0.847 (0.897)	0.713 (0.970)	0.758 (0.898)
0	0.878 (0.658)	0.879 (0.711)	0.816 (0.636)	0.805 (0.680)
	0.930 (0.657)	0.892 (0.691)	0.846 (0.766)	0.810 (0.741)
+0.5	0.744 (0.185)	0.860 (0.408)	0.683 (0.179)	0.791 (0.399)
	1.070 (0.245)	0.913 (0.405)	0.931 (0.311)	0.836 (0.454)
+1	0.507 (0.039)	0.837 (0.219)	0.480 (0.037)	0.770 (0.216)
	1.286 (0.107)	0.931 (0.237)	1.127 (0.129)	0.853 (0.263)

Table 4: Out-of-sample comparisons of *minimum variance* and *minimum LPM* hedge portfolios.

(h^* denotes the minimum variance and the minimum LPM hedge ratios estimated using daily return data over 1988–1993. The minimum LPM h^* values are estimated using different combinations of target return (c) and order of moment (n). The statistics of hedged portfolios (in columns 3–9) are estimated using data over 1994–1999. Panels A and B present the findings for the short hedger and the long hedger respectively.)

Soybean Oil								
(1)	(2) h^*	(3) Mean (μ)	(4) Std. Dev (σ)	(5) μ/σ	(6) Min	(7) Max	(8) 1% Tail	(9) 5% Tail
Panel A: Short Hedge								
Minimum Variance Hedge Portfolio								
	0.994	-0.003%	0.313%	-0.008	-3.667%	1.968%	-1.019%	-0.307%
(c, n)	Minimum LPM Hedge Portfolio							
(-3, 1)	1.031	-0.002%	0.314%	-0.006	-3.731%	2.039%	-0.984%	-0.308%
(-3, 2)	0.964	-0.003%	0.316%	-0.009	-3.615%	1.911%	-1.012%	-0.311%
(-2, 1)	1.439	0.004%	0.603%	0.007	-4.441%	2.818%	-1.620%	-0.930%
(-2, 2)	1.208	0.001%	0.396%	0.002	-4.039%	2.377%	-1.014%	-0.547%
(-1, 1)	1.172	0.000%	0.371%	0.000	-3.976%	2.308%	-0.994%	-0.471%
(-1, 2)	1.272	0.002%	0.446%	0.003	-4.150%	2.499%	-1.185%	-0.646%
(0, 1)	1.009	-0.002%	0.313%	-0.007	-3.693%	1.997%	-1.010%	-0.305%
(0, 2)	1.023	-0.002%	0.313%	-0.007	-3.717%	2.023%	-0.988%	-0.298%
(+1, 1)	1.009	-0.002%	0.313%	-0.007	-3.693%	1.997%	-1.010%	-0.305%
(+1, 2)	1.014	-0.002%	0.313%	-0.007	-3.702%	2.006%	-1.002%	-0.301%
Panel B: Long Hedge								
Minimum Variance Hedge Portfolio								
	0.994	0.003%	0.313%	0.008	-1.968%	3.667%	-0.922%	-0.349%
(c, n)	Minimum LPM Hedge Portfolio							
(-3, 1)	0.367	0.012%	0.815%	0.014	-2.938%	4.028%	-2.153%	-1.213%
(-3, 2)	0.338	0.012%	0.847%	0.014	-3.061%	4.075%	-2.244%	-1.255%
(-2, 1)	0.665	0.007%	0.508%	0.014	-2.241%	3.545%	-1.279%	-0.735%
(-2, 2)	0.489	0.010%	0.684%	0.014	-2.489%	3.830%	-1.780%	-1.049%
(-1, 1)	0.817	0.005%	0.382%	0.013	-2.053%	3.359%	-1.075%	-0.488%
(-1, 2)	0.702	0.007%	0.474%	0.014	-2.195%	3.485%	-1.209%	-0.674%
(0, 1)	1.008	0.002%	0.313%	0.007	-1.995%	3.691%	-0.913%	-0.349%
(0, 2)	0.963	0.003%	0.316%	0.009	-1.909%	3.613%	-0.977%	-0.356%
(+1, 1)	1.034	0.002%	0.315%	0.006	-2.044%	3.736%	-0.901%	-0.348%
(+1, 2)	0.983	0.003%	0.313%	0.009	-1.947%	3.648%	-0.936%	-0.343%

(Table 4 cont'd)

Corn								
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
	h^*	Mean (μ)	Std. Dev (σ)	μ/σ	Min	Max	1% Tail	5% Tail
Panel A: Short Hedge								
Minimum Variance Hedge Portfolio								
	1.044	-0.015%	0.637%	-0.024	-4.743%	3.165%	-2.521%	-0.780%
(c, n)	Minimum LPM Hedge Portfolio							
(-3, 1)	1.638	-0.010%	1.062%	-0.009	-8.210%	5.413%	-3.043%	-1.644%
(-3, 2)	1.637	-0.010%	1.061%	-0.009	-8.203%	5.409%	-3.043%	-1.642%
(-2, 1)	1.371	-0.012%	0.808%	-0.015	-6.349%	4.402%	-2.726%	-1.193%
(-2, 2)	1.466	-0.011%	0.891%	-0.013	-7.011%	4.762%	-2.845%	-1.355%
(-1, 1)	1.135	-0.015%	0.661%	-0.022	-4.843%	3.509%	-2.636%	-0.881%
(-1, 2)	1.276	-0.013%	0.737%	-0.018	-5.687%	4.043%	-2.760%	-1.084%
(0, 1)	1.021	-0.016%	0.634%	-0.025	-4.718%	3.078%	-2.494%	-0.783%
(0, 2)	1.076	-0.015%	0.643%	-0.024	-4.778%	3.286%	-2.562%	-0.813%
(+1, 1)	1.082	-0.015%	0.645%	-0.023	-4.785%	3.308%	-2.569%	-0.820%
(+1, 2)	1.054	-0.015%	0.639%	-0.024	-4.754%	3.202%	-2.534%	-0.776%
Panel B: Long Hedge								
Minimum Variance Hedge Portfolio								
	1.044	0.015%	0.637%	0.024	-3.165%	4.743%	-1.629%	-0.732%
(c, n)	Minimum LPM Hedge Portfolio							
(-3, 1)	0.865	0.017%	0.654%	0.026	-2.839%	4.665%	-1.717%	-0.752%
(-3, 2)	0.904	0.017%	0.643%	0.026	-2.839%	4.668%	-1.691%	-0.734%
(-2, 1)	0.965	0.016%	0.634%	0.026	-2.929%	4.672%	-1.653%	-0.732%
(-2, 2)	0.953	0.016%	0.635%	0.026	-2.902%	4.671%	-1.659%	-0.728%
(-1, 1)	0.942	0.016%	0.636%	0.026	-2.878%	4.671%	-1.664%	-0.730%
(-1, 2)	0.943	0.016%	0.636%	0.026	-2.880%	4.671%	-1.663%	-0.729%
(0, 1)	1.011	0.016%	0.634%	0.025	-3.040%	4.708%	-1.638%	-0.726%
(0, 2)	1.008	0.016%	0.633%	0.025	-3.028%	4.704%	-1.639%	-0.728%
(+1, 1)	1.124	0.015%	0.657%	0.022	-3.467%	4.831%	-1.651%	-0.768%
(+1, 2)	1.014	0.016%	0.634%	0.025	-3.051%	4.711%	-1.637%	-0.725%

(Table 4 cont'd)

Cotton								
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
	h^*	Mean (μ)	Std. Dev (σ)	μ/σ	Min	Max	1% Tail	5% Tail
Panel A: Short Hedge								
Minimum Variance Hedge Portfolio								
	0.912	-0.002%	0.702%	-0.003	-5.387%	4.941%	-2.198%	-1.017%
(c, n)	Minimum LPM Hedge Portfolio							
(-3, 1)	0.886	-0.001%	0.703%	-0.002	-5.398%	4.917%	-2.218%	-1.000%
(-3, 2)	0.942	-0.002%	0.703%	-0.003	-5.375%	4.970%	-2.262%	-1.018%
(-2, 1)	0.713	0.002%	0.749%	0.003	-5.472%	4.754%	-2.290%	-1.050%
(-2, 2)	0.775	0.001%	0.725%	0.001	-5.445%	4.813%	-2.169%	-1.026%
(-1, 1)	0.660	0.003%	0.775%	0.004	-5.494%	4.704%	-2.375%	-1.136%
(-1, 2)	0.656	0.004%	0.777%	0.005	-5.496%	4.701%	-2.380%	-1.133%
(0, 1)	1.053	-0.005%	0.722%	-0.006	-5.328%	5.190%	-2.516%	-1.038%
(0, 2)	0.909	-0.002%	0.702%	-0.002	-5.389%	4.939%	-2.197%	-1.015%
(+1, 1)	0.675	0.003%	0.767%	0.004	-5.488%	4.718%	-2.354%	-1.112%
(+1, 2)	0.934	-0.002%	0.703%	-0.003	-5.378%	4.962%	-2.244%	-1.024%
Panel B: Long Hedge								
Minimum Variance Hedge Portfolio								
	0.912	0.002%	0.702%	0.003	-4.941%	5.387%	-2.400%	-0.895%
(c, n)	Minimum LPM Hedge Portfolio							
(-3, 1)	0.334	-0.010%	1.019%	-0.010	-4.398%	5.632%	-2.526%	-1.638%
(-3, 2)	0.657	-0.004%	0.776%	-0.005	-4.702%	5.496%	-2.343%	-1.022%
(-2, 1)	0.528	-0.006%	0.858%	-0.007	-4.580%	5.550%	-2.277%	-1.261%
(-2, 2)	0.514	-0.006%	0.868%	-0.007	-4.567%	5.556%	-2.306%	-1.291%
(-1, 1)	0.661	-0.003%	0.774%	-0.004	-4.705%	5.494%	-2.346%	-1.025%
(-1, 2)	0.648	-0.004%	0.781%	-0.005	-4.693%	5.499%	-2.336%	-1.021%
(0, 1)	1.053	0.005%	0.722%	0.006	-5.190%	5.328%	-2.514%	-0.957%
(0, 2)	0.915	0.002%	0.702%	0.003	-4.944%	5.386%	-2.399%	-0.899%
(+1, 1)	0.647	-0.004%	0.782%	-0.005	-4.692%	5.500%	-2.335%	-1.019%
(+1, 2)	0.927	0.002%	0.702%	0.003	-4.955%	5.381%	-2.396%	-0.905%

(Table 4 cont'd)

Crude Oil								
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
	h^*	Mean (μ)	Std. Dev (σ)	μ/σ	Min	Max	1% Tail	5% Tail
Panel A: Short Hedge								
Minimum Variance Hedge Portfolio								
	0.970	-0.055%	1.251%	-0.044	-13.356%	16.830%	-4.300%	-1.371%
(c, n)	Minimum LPM Hedge Portfolio							
(-3, 1)	1.073	-0.061%	1.271%	-0.048	-13.121%	16.674%	-4.405%	-1.394%
(-3, 2)	1.240	-0.069%	1.366%	-0.051	-12.741%	16.421%	-4.957%	-1.598%
(-2, 1)	0.870	-0.050%	1.263%	-0.040	-13.583%	16.981%	-4.170%	-1.433%
(-2, 2)	0.944	-0.054%	1.251%	-0.043	-13.415%	16.869%	-4.321%	-1.373%
(-1, 1)	1.042	-0.059%	1.262%	-0.047	-13.192%	16.721%	-4.298%	-1.357%
(-1, 2)	0.933	-0.053%	1.252%	-0.043	-13.440%	16.886%	-4.301%	-1.382%
(0, 1)	1.001	-0.057%	1.254%	-0.045	-13.285%	16.783%	-4.288%	-1.317%
(0, 2)	0.979	-0.056%	1.252%	-0.045	-13.335%	16.816%	-4.293%	-1.357%
(+1, 1)	0.873	-0.050%	1.262%	-0.040	-13.577%	16.976%	-4.176%	-1.431%
(+1, 2)	0.971	-0.055%	1.251%	-0.044	-13.353%	16.828%	-4.299%	-1.370%
Panel B: Long Hedge								
Minimum Variance Hedge Portfolio								
	0.970	0.055%	1.251%	0.044	-16.830%	13.356%	-2.571%	-1.017%
(c, n)	Minimum LPM Hedge Portfolio							
(-3, 1)	0.963	0.055%	1.251%	0.044	-16.840%	13.372%	-2.557%	-1.022%
(-3, 2)	0.963	0.055%	1.251%	0.044	-16.840%	13.372%	-2.557%	-1.022%
(-2, 1)	0.872	0.050%	1.262%	0.040	-16.978%	13.579%	-2.403%	-1.069%
(-2, 2)	1.033	0.059%	1.260%	0.046	-16.734%	13.212%	-2.786%	-1.096%
(-1, 1)	0.887	0.051%	1.259%	0.041	-16.955%	13.545%	-2.409%	-1.060%
(-1, 2)	0.907	0.052%	1.255%	0.041	-16.925%	13.499%	-2.448%	-1.050%
(0, 1)	1.001	0.057%	1.254%	0.045	-16.783%	13.285%	-2.677%	-1.032%
(0, 2)	0.958	0.055%	1.251%	0.044	-16.848%	13.383%	-2.547%	-1.010%
(+1, 1)	1.042	0.059%	1.262%	0.047	-16.721%	13.192%	-2.784%	-1.109%
(+1, 2)	0.974	0.055%	1.252%	0.044	-16.824%	13.347%	-2.585%	-1.015%

(Table 4 cont'd)

Wheat								
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
	h^*	Mean (μ)	Std. Dev (σ)	μ/σ	Min	Max	1% Tail	5% Tail
Panel A: Short Hedge								
Minimum Variance Hedge Portfolio								
	0.619	-0.053%	1.188%	-0.045	-7.508%	7.083%	-3.225%	-1.838%
(c, n)	Minimum LPM Hedge Portfolio							
(-3, 1)	0.592	-0.053%	1.186%	-0.044	-7.603%	7.038%	-3.207%	-1.821%
(-3, 2)	0.533	-0.052%	1.185%	-0.044	-7.811%	6.941%	-3.159%	-1.840%
(-2, 1)	0.719	-0.054%	1.206%	-0.045	-7.155%	7.248%	-3.290%	-1.907%
(-2, 2)	0.690	-0.054%	1.199%	-0.045	-7.257%	7.200%	-3.271%	-1.899%
(-1, 1)	0.509	-0.051%	1.186%	-0.043	-7.895%	6.901%	-3.139%	-1.844%
(-1, 2)	0.640	-0.053%	1.191%	-0.045	-7.434%	7.117%	-3.238%	-1.863%
(0, 1)	0.564	-0.052%	1.185%	-0.044	-7.701%	6.992%	-3.185%	-1.846%
(0, 2)	0.582	-0.053%	1.185%	-0.044	-7.638%	7.022%	-3.200%	-1.823%
(+1, 1)	0.552	-0.052%	1.185%	-0.044	-7.744%	6.972%	-3.175%	-1.837%
(+1, 2)	0.577	-0.052%	1.185%	-0.044	-7.656%	7.014%	-3.196%	-1.827%
Panel B: Long Hedge								
Minimum Variance Hedge Portfolio								
	0.619	0.053%	1.188%	0.045	-7.083%	7.508%	-3.028%	-1.748%
(c, n)	Minimum LPM Hedge Portfolio							
(-3, 1)	0.592	0.053%	1.186%	0.044	-7.038%	7.603%	-3.001%	-1.756%
(-3, 2)	0.593	0.053%	1.186%	0.044	-7.040%	7.599%	-3.002%	-1.757%
(-2, 1)	0.749	0.055%	1.215%	0.045	-7.297%	7.049%	-3.220%	-1.777%
(-2, 2)	0.670	0.054%	1.195%	0.045	-7.167%	7.328%	-3.081%	-1.740%
(-1, 1)	0.667	0.054%	1.195%	0.045	-7.162%	7.338%	-3.078%	-1.741%
(-1, 2)	0.680	0.054%	1.197%	0.045	-7.183%	7.293%	-3.092%	-1.735%
(0, 1)	0.623	0.053%	1.188%	0.045	-7.089%	7.493%	-3.032%	-1.750%
(0, 2)	0.656	0.054%	1.193%	0.045	-7.144%	7.377%	-3.066%	-1.747%
(+1, 1)	0.689	0.054%	1.199%	0.045	-7.198%	7.261%	-3.102%	-1.743%
(+1, 2)	0.642	0.053%	1.191%	0.045	-7.121%	7.426%	-3.050%	-1.745%

(Table 4 cont'd)

Japanese Yen								
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
	h^*	Mean (μ)	Std. Dev (σ)	μ/σ	Min	Max	1% Tail	5% Tail
Panel A: Short Hedge								
Minimum Variance Hedge Portfolio								
	0.967	0.016%	0.167%	0.096	-1.020%	0.961%	-0.417%	-0.246%
(c, n)	Minimum LPM Hedge Portfolio							
(-3, 1)	0.494	-0.003%	0.371%	-0.009	-1.645%	2.331%	-0.857%	-0.577%
(-3, 2)	0.494	-0.003%	0.371%	-0.009	-1.645%	2.331%	-0.857%	-0.577%
(-2, 1)	0.934	0.015%	0.166%	0.088	-1.050%	0.844%	-0.413%	-0.248%
(-2, 2)	0.934	0.015%	0.166%	0.088	-1.050%	0.844%	-0.413%	-0.248%
(-1, 1)	0.881	0.012%	0.172%	0.072	-1.099%	0.655%	-0.412%	-0.266%
(-1, 2)	0.883	0.013%	0.172%	0.073	-1.097%	0.662%	-0.412%	-0.264%
(0, 1)	0.969	0.016%	0.167%	0.096	-1.018%	0.968%	-0.419%	-0.247%
(0, 2)	0.959	0.016%	0.166%	0.094	-1.027%	0.933%	-0.414%	-0.245%
(+1, 1)	1.018	0.018%	0.174%	0.104	-0.973%	1.142%	-0.420%	-0.247%
(+1, 2)	0.965	0.016%	0.166%	0.096	-1.022%	0.954%	-0.416%	-0.247%
Panel B: Long Hedge								
Minimum Variance Hedge Portfolio								
	0.967	-0.016%	0.167%	-0.096	-0.961%	1.020%	-0.466%	-0.277%
(c, n)	Minimum LPM Hedge Portfolio							
(-3, 1)	0.364	0.009%	0.459%	0.019	-2.949%	2.056%	-1.318%	-0.750%
(-3, 2)	0.364	0.009%	0.459%	0.019	-2.949%	2.056%	-1.318%	-0.750%
(-2, 1)	0.636	-0.002%	0.281%	-0.008	-1.656%	1.325%	-0.768%	-0.443%
(-2, 2)	0.636	-0.002%	0.281%	-0.008	-1.656%	1.325%	-0.768%	-0.443%
(-1, 1)	1.040	-0.019%	0.180%	-0.105	-1.220%	0.953%	-0.513%	-0.305%
(-1, 2)	1.005	-0.018%	0.172%	-0.102	-1.096%	0.985%	-0.489%	-0.285%
(0, 1)	0.972	-0.016%	0.167%	-0.097	-0.979%	1.015%	-0.464%	-0.277%
(0, 2)	0.974	-0.016%	0.167%	-0.097	-0.986%	1.013%	-0.465%	-0.278%
(+1, 1)	0.883	-0.013%	0.172%	-0.073	-0.662%	1.097%	-0.516%	-0.285%
(+1, 2)	0.971	-0.016%	0.167%	-0.097	-0.975%	1.016%	-0.464%	-0.277%

(Table 4 cont'd)

British Pound								
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
	h^*	Mean (μ)	Std. Dev (σ)	μ/σ	Min	Max	1% Tail	5% Tail
Panel A: Short Hedge								
Minimum Variance Hedge Portfolio								
	0.951	-0.003%	0.116%	-0.024	-0.673%	0.500%	-0.278%	-0.188%
(c, n)	Minimum LPM Hedge Portfolio							
(-3, 1)	0.446	0.004%	0.274%	0.013	-1.342%	1.239%	-0.721%	-0.441%
(-3, 2)	0.446	0.004%	0.274%	0.013	-1.342%	1.239%	-0.721%	-0.441%
(-2, 1)	0.796	-0.001%	0.137%	-0.006	-0.636%	0.570%	-0.349%	-0.226%
(-2, 2)	0.796	-0.001%	0.137%	-0.006	-0.636%	0.570%	-0.349%	-0.226%
(-1, 1)	1.001	-0.003%	0.120%	-0.028	-0.685%	0.595%	-0.313%	-0.192%
(-1, 2)	1.046	-0.004%	0.128%	-0.031	-0.695%	0.721%	-0.317%	-0.202%
(0, 1)	0.956	-0.003%	0.116%	-0.025	-0.674%	0.502%	-0.289%	-0.187%
(0, 2)	0.952	-0.003%	0.116%	-0.024	-0.673%	0.501%	-0.280%	-0.188%
(+1, 1)	0.787	-0.001%	0.139%	-0.005	-0.634%	0.584%	-0.357%	-0.228%
(+1, 2)	0.949	-0.003%	0.116%	-0.024	-0.672%	0.499%	-0.280%	-0.188%
Panel B: Long Hedge								
Minimum Variance Hedge Portfolio								
	0.951	0.003%	0.116%	0.024	-0.500%	0.673%	-0.301%	-0.180%
(c, n)	Minimum LPM Hedge Portfolio							
(-3, 1)	0.272	-0.006%	0.355%	-0.016	-1.762%	1.709%	-0.943%	-0.622%
(-3, 2)	0.272	-0.006%	0.355%	-0.016	-1.762%	1.709%	-0.943%	-0.622%
(-2, 1)	0.530	-0.002%	0.236%	-0.010	-0.987%	1.165%	-0.634%	-0.397%
(-2, 2)	0.530	-0.002%	0.236%	-0.010	-0.987%	1.165%	-0.634%	-0.397%
(-1, 1)	0.966	0.003%	0.117%	0.026	-0.507%	0.676%	-0.307%	-0.180%
(-1, 2)	0.980	0.003%	0.118%	0.027	-0.536%	0.680%	-0.299%	-0.180%
(0, 1)	0.957	0.003%	0.116%	0.025	-0.503%	0.674%	-0.306%	-0.180%
(0, 2)	0.950	0.003%	0.116%	0.024	-0.500%	0.673%	-0.301%	-0.180%
(+1, 1)	1.043	0.004%	0.127%	0.031	-0.713%	0.695%	-0.305%	-0.193%
(+1, 2)	0.952	0.003%	0.116%	0.024	-0.501%	0.673%	-0.302%	-0.180%

(Table 4 cont'd)

Deutsche Mark								
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
	h^*	Mean (μ)	Std. Dev (σ)	μ/σ	Min	Max	1% Tail	5% Tail
Panel A: Short Hedge								
Minimum Variance Hedge Portfolio								
	0.963	0.005%	0.140%	0.034	-0.789%	0.673%	-0.357%	-0.213%
(c, n)	Minimum LPM Hedge Portfolio							
(-3, 1)	0.382	0.000%	0.385%	-0.001	-1.943%	1.682%	-0.942%	-0.642%
(-3, 2)	0.382	0.000%	0.385%	-0.001	-1.943%	1.682%	-0.942%	-0.642%
(-2, 1)	0.610	0.002%	0.258%	0.007	-1.208%	1.037%	-0.612%	-0.436%
(-2, 2)	0.610	0.002%	0.258%	0.007	-1.208%	1.037%	-0.612%	-0.436%
(-1, 1)	0.893	0.004%	0.146%	0.029	-0.794%	0.638%	-0.392%	-0.221%
(-1, 2)	0.885	0.004%	0.147%	0.028	-0.794%	0.639%	-0.392%	-0.220%
(0, 1)	0.966	0.005%	0.140%	0.034	-0.788%	0.677%	-0.358%	-0.213%
(0, 2)	0.957	0.005%	0.140%	0.034	-0.789%	0.663%	-0.357%	-0.213%
(+1, 1)	1.097	0.006%	0.163%	0.036	-0.872%	0.880%	-0.503%	-0.269%
(+1, 2)	0.963	0.005%	0.140%	0.034	-0.789%	0.673%	-0.357%	-0.213%
Panel B: Long Hedge								
Minimum Variance Hedge Portfolio								
	0.963	-0.005%	0.140%	-0.034	-0.673%	0.789%	-0.363%	-0.212%
(c, n)	Minimum LPM Hedge Portfolio							
(-3, 1)	0.310	0.001%	0.427%	0.002	-1.903%	2.175%	-1.142%	-0.689%
(-3, 2)	0.310	0.001%	0.427%	0.002	-1.903%	2.175%	-1.142%	-0.689%
(-2, 1)	0.718	-0.003%	0.205%	-0.013	-0.763%	0.860%	-0.502%	-0.341%
(-2, 2)	0.718	-0.003%	0.205%	-0.013	-0.763%	0.860%	-0.502%	-0.341%
(-1, 1)	1.094	-0.006%	0.162%	-0.036	-0.875%	0.867%	-0.395%	-0.255%
(-1, 2)	1.121	-0.006%	0.172%	-0.036	-0.917%	0.912%	-0.422%	-0.267%
(0, 1)	0.965	-0.005%	0.140%	-0.034	-0.676%	0.788%	-0.364%	-0.211%
(0, 2)	0.968	-0.005%	0.140%	-0.035	-0.680%	0.788%	-0.365%	-0.210%
(+1, 1)	0.837	-0.004%	0.159%	-0.023	-0.646%	0.798%	-0.419%	-0.250%
(+1, 2)	0.962	-0.005%	0.140%	-0.034	-0.671%	0.789%	-0.362%	-0.212%

(Table 4 cont'd)

S&P 500								
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
	h^*	Mean (μ)	Std. Dev (σ)	μ/σ	Min	Max	1% Tail	5% Tail
Panel A: Short Hedge								
Minimum Variance Hedge Portfolio								
	0.884	0.020%	0.196%	0.102	-0.819%	0.800%	-0.537%	-0.301%
(c, n)	Minimum LPM Hedge Portfolio							
(-3, 1)	0.514	0.044%	0.377%	0.116	-3.153%	2.101%	-0.966%	-0.548%
(-3, 2)	0.514	0.044%	0.377%	0.116	-3.153%	2.101%	-0.966%	-0.548%
(-2, 1)	0.610	0.038%	0.308%	0.122	-2.413%	1.562%	-0.744%	-0.443%
(-2, 2)	0.610	0.038%	0.308%	0.122	-2.413%	1.562%	-0.744%	-0.443%
(-1, 1)	0.783	0.026%	0.214%	0.124	-1.080%	0.845%	-0.535%	-0.329%
(-1, 2)	0.770	0.027%	0.218%	0.125	-1.180%	0.866%	-0.537%	-0.333%
(0, 1)	0.903	0.019%	0.197%	0.095	-0.863%	0.834%	-0.541%	-0.305%
(0, 2)	0.888	0.020%	0.196%	0.100	-0.828%	0.808%	-0.538%	-0.300%
(+1, 1)	0.592	0.039%	0.320%	0.121	-2.552%	1.663%	-0.797%	-0.452%
(+1, 2)	0.857	0.022%	0.197%	0.110	-0.780%	0.752%	-0.531%	-0.311%
Panel B: Long Hedge								
Minimum Variance Hedge Portfolio								
	0.884	-0.020%	0.196%	-0.102	-0.800%	0.819%	-0.528%	-0.331%
(c, n)	Minimum LPM Hedge Portfolio							
(-3, 1)	0.256	-0.060%	0.582%	-0.103	-3.551%	5.140%	-1.425%	-0.960%
(-3, 2)	0.256	-0.060%	0.582%	-0.103	-3.551%	5.140%	-1.425%	-0.960%
(-2, 1)	0.461	-0.047%	0.417%	-0.113	-2.399%	3.561%	-1.042%	-0.707%
(-2, 2)	0.461	-0.047%	0.417%	-0.113	-2.399%	3.561%	-1.042%	-0.707%
(-1, 1)	0.824	-0.024%	0.202%	-0.118	-0.778%	0.768%	-0.519%	-0.351%
(-1, 2)	0.830	-0.023%	0.201%	-0.117	-0.768%	0.770%	-0.519%	-0.339%
(0, 1)	0.924	-0.017%	0.200%	-0.087	-0.872%	0.911%	-0.510%	-0.327%
(0, 2)	0.879	-0.020%	0.196%	-0.103	-0.791%	0.808%	-0.525%	-0.332%
(+1, 1)	1.188	0.000%	0.334%	-0.001	-2.040%	1.685%	-0.937%	-0.569%
(+1, 2)	0.914	-0.018%	0.198%	-0.091	-0.854%	0.888%	-0.508%	-0.322%

(Table 4 cont'd)

				NYSE 100				
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
	h^*	Mean (μ)	Std. Dev (σ)	μ/σ	Min	Max	1% Tail	5% Tail
				Panel A: Short Hedge				
				Minimum Variance Hedge Portfolio				
	0.797	0.022%	0.199%	0.113	-0.938%	0.697%	-0.579%	-0.320%
(c, n)				Minimum LPM Hedge Portfolio				
(-3, 1)	0.466	0.042%	0.362%	0.115	-2.884%	1.401%	-0.986%	-0.546%
(-3, 2)	0.466	0.042%	0.362%	0.115	-2.884%	1.401%	-0.986%	-0.546%
(-2, 1)	0.555	0.037%	0.303%	0.121	-2.138%	1.172%	-0.819%	-0.451%
(-2, 2)	0.555	0.037%	0.303%	0.121	-2.138%	1.172%	-0.819%	-0.451%
(-1, 1)	0.726	0.027%	0.216%	0.124	-1.021%	0.806%	-0.590%	-0.348%
(-1, 2)	0.656	0.031%	0.245%	0.125	-1.291%	0.956%	-0.633%	-0.370%
(0, 1)	0.817	0.021%	0.198%	0.108	-0.956%	0.689%	-0.595%	-0.312%
(0, 2)	0.800	0.022%	0.199%	0.112	-0.938%	0.695%	-0.578%	-0.317%
(+1, 1)	0.529	0.038%	0.320%	0.119	-2.356%	1.228%	-0.874%	-0.483%
(+1, 2)	0.774	0.024%	0.203%	0.117	-0.965%	0.716%	-0.574%	-0.331%
				Panel B: Long Hedge				
				Minimum Variance Hedge Portfolio				
	0.797	-0.022%	0.199%	-0.113	-0.697%	0.938%	-0.507%	-0.314%
(c, n)				Minimum LPM Hedge Portfolio				
(-3, 1)	0.263	-0.054%	0.513%	-0.104	-2.581%	4.587%	-1.277%	-0.834%
(-3, 2)	0.263	-0.054%	0.513%	-0.104	-2.581%	4.587%	-1.277%	-0.834%
(-2, 1)	0.460	-0.042%	0.367%	-0.115	-1.436%	2.935%	-0.926%	-0.623%
(-2, 2)	0.460	-0.042%	0.367%	-0.115	-1.436%	2.935%	-0.926%	-0.623%
(-1, 1)	0.715	-0.027%	0.220%	-0.124	-0.830%	1.034%	-0.531%	-0.356%
(-1, 2)	0.724	-0.027%	0.216%	-0.124	-0.810%	1.023%	-0.522%	-0.353%
(0, 1)	0.841	-0.020%	0.198%	-0.101	-0.686%	0.983%	-0.522%	-0.309%
(0, 2)	0.792	-0.023%	0.200%	-0.114	-0.699%	0.944%	-0.506%	-0.316%
(+1, 1)	1.072	-0.006%	0.283%	-0.023	-2.199%	2.120%	-0.767%	-0.442%
(+1, 2)	0.829	-0.021%	0.197%	-0.104	-0.688%	0.970%	-0.529%	-0.311%