Math 464, Differential Equations II

(Adopted, Fall 2006)

Course Description: Introduction to partial differential equations, heat equation, wave equation, Laplace's equation, Fourier series, Fourier transform, method of seperation of variables.

Prerequisites: Math 250 and Math 305, Math 321.

Contents:

1. Introduction to partial differential equations. Classification.
2. The one-dimensional heat equation.
   (1) Mathematical model.
   (2) Initial and boundary value problems. Three different boundary conditions.
   (3) The Maximum Principle and uniqueness of the solution.
   (4) Steady-state solutions.
3. The one-dimensional wave equation.
   (1) Mathematical model.
   (2) Initial and boundary value problems. Three different boundary conditions
   (3) Conservation of energy and uniqueness of the solution.
   (4) Method of Characteristics for first order partial differential equations.
   (5) D'Alemberts' solutions to the one-dimensional wave equation.
4. The essentials of Fourier series
   (1) Orthogonality and inner product.
   (2) The function space of piecewise smooth functions.
   (3) Even and odd functions and Fourier series.
   (4) Fourier series representations of piecewise smooth functions.
   (5) The Fourier convergence theorem.
   (6) Differentiation and integration of Fourier series.
5. The method of separation of variables: homogeneous problem.
   (1) Solve the heat equation.
   (2) Solve the wave equation.
   (3) Solve Laplace' equation.
6. Fourier integrals and transform methods.
   (1) Fourier integral.
   (2) The Fourier transformation,
   (3) Fourier transform solution method of partial differential equations.
7. Optional subject.
   (1) The Sturm-Liouville eigenvalue problem.
   (2) Separation of variables: the nonhomogeneous problem.

Textbook: Partial differential equations with Fourier series and boundary value problems, 2nd edition, by Nakhi, H. Asmar, Prentice Hall, 2005

Sections to be covered in the textbook:

1. Chapter 1: sections 1.1, 1.2. A preview of Applications and Techniques. The method of characteristic should be covered with supplement. Exercise 1.1: 5-14 must be given.
2. Chapter 2. Fourier series: 2.1-2.7. Results of sections 2.8, 2.9 and 2.10 must be given, but the proofs may be outlined.
3. Chapter 3. Partial differential equations in rectangular coordinates. 3.1-3.11. The proof of the Maximum principle and the uniqueness of solutions should be given.
4. Selected sections from 4.1 to 4.4: cylindrical coordinates.
5. Chapter 7: 7.1-7.3. The Fourier transform and its applications.
6. Optional: chapter 6: 6.1-6.3: Sturm-Liouville theory.