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WIESŁAW ŻELAZKO

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1. Life of Wiesław Żelazko Professor Wiesław Żelazko was born in Łódź, central Poland, on February 16, 1933, the son of Zofia and Władyslaw. His father was an educator and social worker but the family roots can be traced to rural areas around Łódź and the iron smith tradition - hence the name Żelazko (iron in polish). His early education and the family life was interrupted by the outbreak of the Second World War and the German occupation of Poland, he spent most that period in Piotrków. In 1951 he graduated from high school and as one of the winners of the Second Mathematical Olympiad was admitted to the Warsaw University.

While the economic and political situations in Poland during that time was difficult the scientific life, especially in mathematics flourished. A large number of prominent mathematicians were active in Warsaw and other cities and enjoyed relative freedom of research. There were several weekly seminars on various topics; some, like the *Tuesday Functional Analysis Seminar*, continue till today. While the research activities in most other areas were severely restricted by the communist government, or by the lack of equipment, mathematics was perceived by the authorities as unrelated to the real world and consequently was often left alone.

Żelazko got his M.Sc. under the supervision of Prof. Roman Sikorski in 1955 and for the next two years worked as a Teaching Assistant for the Institute of Mathematics, Warsaw University. In 1957 he moved to the Institute of Mathematics of the Polish Academy of Sciences (IMPAN) where he remains till today always however maintaining close contacts with the Warsaw University. In fact these two institutions located in a close proximity in Warsaw while having different educational goals and obligations often work as one large research group.

Żelazko obtained his doctorate in January of 1960 under the supervision of Prof.

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Stanisław Mazur. He become an adiunkt (1960), docent (1965), extraordinary professor (1971), and ordinary professor (1976). While working mostly in Warsaw he travel extensively lecturing in over 40 countries and maintained mathematical contact worldwide. His longer visits (6 months to a year) included Moscow State University, Yale University, Aarhus University, University of Kansas; he held shorter visiting positions at the University of Washington (Seattle), Université Paris-Sud, University of California Berkeley, University of Newcastle-upon-Tyne, Universidad de los Andes (Venezuela), Universidad National Autónoma de México, and many other.

Żelazko supervised well over 100 Master thesis and nine Ph.D. dissertations. The list of his doctorate students includes Piotr Uss, Czesław Matyszczyk, Zbigniew Słodkowski, Tomasz Müldner, Nguyen Van Khue, Jaroslav Zemánek, Ewa Ligocka, Andrzej Sołtysiak, and Krzysztof Jarosz. He also devoted a lot of time and energy to administrative and editorial work. For the last 35 years he has been an associate editor and later the Managing Editor of *Studia Mathematica*, he has been associate editor of *Dissertations Mathematica*, chair of the Editorial Committee of *Delta*, member of the Editorial Committees of *Gradient* (student/teacher magazine), and *Commentationes Mathematicae*; since 1972, for almost 20 years he worked as a deputy director of IMPAN.

2. Research of Wiesław Żelazko The research of Wiesław Żelazko spans almost half of a century and well over 100 papers, many of which started a new line of research. Here we will concentrate only on few most important aspects of his research, any such selection is necessarily very subjective.

2.1. Early papers The first paper by Żelazko ([1957a]) dealt with divisors of zero and the second one ([1957b]), coauthored with A. Białynicki-Birula, with multiplicative functionals. He returned to both topics several times in the following decades. In [1957b] he proved that in the Cartesian product $\prod_{t\in T} R_t$ of algebras R_t all multiplicative functionals are given in a trivial way, that is as a composition of a projection on some R_{t_0} with a multiplicative functional on R_{t_0} if and only if the index set T is of none measurable cardinality.

Other early papers leading to his Ph.D. in early 1960 concern primarily division algebras. One of the main theorems from this period states that a complete p-normed commutative division algebra over the field $\mathbb{C}$ of complex numbers is one-dimensional ([1960a]). The proof introduced new methods and did not use the analytic function theory; indeed, the local compactness, connectivity and the fact that $\mathbb{C}$ is algebraically closed are the only properties of $\mathbb{C}$ used. That result is further generalized to show that a Hausdorff topological division algebra (only continuity of multiplication in each variable separately is assumed) over the complex numbers that has a bounded neighborhood of zero is one-dimensional. Many other basic theorems on normed and Banach algebras are consequently generalized to p-normed and complete p-normed algebras. Extending a well-known result on locally convex spaces Żelazko showed that a Hausdorff topological algebra whose topology is given by a family of submultiplicative p-seminorms (p variable) is topologically isomorphic to a subdirect product of a family of p-normed algebras.

2.2. Divisors of zero and generalized topological divisors of zero Żelazko's intense interest with divisors of zero and later generalized topological divisors started with his very first paper ([1957a]) written just after his M.Sc. degree. He proved there that any convolution Banach algebra $L_1(G)$ must contain divisors of zero.

The classical theorem of Shilov asserts that a real Banach algebra devoid of nontrivial topological divisors of zero is isomorphic to the field of real numbers, complex numbers, or quaternions. In seeking generalizations to arbitrary topological algebras Żelazko in the sixties introduced the concept of generalized topological divisors of zero and proved that an m-convex topological algebra without such divisors must be trivial, that is equal to the field of complex numbers ([1966]). We say that a topological algebra $\mathcal{A}$ has generalized topological divisors of zero if there is a pair of subsets P, Q of $\mathcal{A}$ such that zero is in the closure of PQ, but neither in the closure of P nor in the closure of Q. Soon he extended the result showing that a real p-normed or m-convex algebra either contains generalized topological divisors of zero or is isomorphic to one of the three finite-dimensional real division algebras [1967a],[1967b]. It took almost two decades to show that the result is not true for B_0-algebras - in 1985 H. Arizmendi and W. Żelazko constructed a complex commutative B_0-algebra, not equal to $\mathbb{C}$, but without generalized topological divisors of zero [1985b]. The example is the set of power series $x = \sum_{i=0}^{\infty} \zeta_i t^i$ such that $\|x\|_n = \sum_{i=0}^{\infty} a_i^{(n)} |\zeta_i| < \infty$ $(n \in \mathbb{N})$, where $(a_i^{(n)}:\ i, n \in \mathbb{N})$ is a certain set of positive real numbers.

The most important paper concerning generalized topological divisors of zero was published in 1987 ([1987c]). Żelazko proved there that in any complete unital topological algebra without generalized topological divisors of zero the spectrum of any element, other then a multiple of identity, is empty or is equal to the entire scalar field. He also showed that any complex m-pseudoconvex algebra has generalized topological divisors of zero or is equal to the complex field. However the most important result of that paper stated that for a real unital topological algebra $\mathcal{A}$ without generalized topological divisors of zero the group $G(\mathcal{A})$ of invertible elements must be isomorphic to one of three multiplicative groups: $\mathbb{R}\backslash\{0\}$, $\mathbb{C}\backslash\{0\}$, $\mathbb{H}\backslash\{0\}$. An important deduction that emerges from the above result is that in a complex topological algebra without generalized topological divisors of zero the group $G(\mathcal{A})$ contains only multiples of identity. The converse result is false even for commutative unital B_0 algebras [1987c]. For further extensions one may consult a survey article [1988a].

2.3. Metric generalizations of Banach algebras In 1965 Żelazko published the first systematic survey of metric generalizations of Banach algebras, based on his lectures given at Yale University in 1963/64 ([1965a]). It contained many of his earlier results with some new theorems, some results of Arens, Michael, Williamson and others as well as several open problems. The first chapter contains mainly the theory of locally bounded complete metric algebras. It is shown that most important properties of Banach algebras are also true for p-normed algebras. The second chapter deals with F-algebras and topological division algebras, the third one with B_0-algebras, and the remaining part with the entire functions operating on topological algebras. This publication was a major part of his habilitation in 1965. Few years later, after a serious of lectures at the University of

Aarhus in 1969/70, the survey was revised, greatly enlarged and published in the Lecture Notes Series in Aarhus ([1971c]).

2.4. Entire functions operating on topological algebras If $\mathcal{A}$ is a complex Banach algebra and $\varphi(z) = \sum_{n=0}^{\infty} a_n z^n$ an entire function then φ operates on $\mathcal{A}$, that is the series $\sum_{n=0}^{\infty} a_n x^n$ is convergent for any $x \in \mathcal{A}$. The statement is obvious and can also be easily extended to m-convex algebras. Clearly the submultiplicativity of the seminorms is essential in that simple proof but more surprisingly it is necessary. In 1962 Żelazko together with B. Mitiagin and S. Rolewicz proved that if all entire functions operate on a given commutative B_0 algebra then it must be m-convex. On the other hand a single entire function is never enough for such conclusion: for any entire function φ there does exist a B_0-algebra R_φ such that φ operates on R_ϕ, but R_ϕ is not m-convex ([1962b]). It is again surprising that there is a noncommutative B_0 algebra on which all entire functions operate but which is not m-convex; of course all commutative subalgebras must be m-convex, such example was however discovered much later ([1994c]). The subject of operating function was also carefully discussed in [1965a] and [1971c]

2.5. Gleason-Kahane-Żelazko Theorem 1968 was a particularly important year in the mathematical life of W. Żelazko, that year he published the proofs of now classical Gleason-Kahane-Żelazko Theorem ([1968a],[1968b]) and the Hirschfeld-Żelazko Theorem ([1968d]) as well as his influential book *Banach Algebras* ([1968e]).

THEOREM 1 (G-K-Ż). *If F is a linear functional on a complex unital Banach algebra $\mathcal{A}$, such that*

$$F(x) \neq 0, \qquad \textit{for } x \in \mathcal{A}^{-1}, \tag{1}$$

then $F/F(e)$ is multiplicative.

The Theorem was proved in the commutative case independently, and surprisingly using the same method, by J.P. Kahane and W. Żelazko ([1968a]) and by A. M. Gleason ([9]). The none commutative case was settled in the same year by W. Żelazko ([1968b]). Since then the result has been phrased in many different ways, generalized in many directions, and influenced a large volume of research. It was immediately noticed that it can be easily generalized to maps between commutative Banach algebras:

THEOREM 2 (G-K-Ż). *If T is a linear map from a complex unital Banach algebra $\mathcal{A}$ into a complex unital commutative semisimple Banach algebra $\mathcal{B}$ such that $\sigma(T(x)) \subset \sigma(x)$, for $x \in \mathcal{A}$, then T is multiplicative.*

The obvious and important problem - how far this version of the Theorem could be extended to other algebras is still not completely settled in spite of many important discoveries, some of them very recent.

THEOREM 3 (B. Aupetit, [3]). *If T is a surjective linear map between complex unital semisimple von Neumann algebras such that $\sigma(T(x)) = \sigma(x)$, for all x, then T is a Jordan isomorphism.*

The result fails in general without the assumption the algebras are semisimple or that T is surjective; it is not known if it holds for all C^*-algebras. A reader may consult survey articles [2] or [12] for more information and further references on this subject.

The Gleason-Kahane-Żelazko Theorem can be also phrased as follows.

THEOREM 4. *Let $\mathcal{A}$ be a complex unital Banach function algebra on a compact set X and let M be a codimension one subspace of $\mathcal{A}$. Then*

$$(\forall f \in M \ \exists x \in X \quad f(x) = 0) \quad \Rightarrow \quad (\exists x \in X \ \forall f \in M \quad f(x) = 0).$$

Almost immediately after the publication of the original paper by Kahane and Żelazko the question was raised if the assumption above that $\mathrm{codim} M = 1$ is essential. It is not very difficult to show that the theorem fails for general subspaces ([12]) but the question whether $\mathrm{codim} M < \infty$ is sufficient is still open after more then three decades. It is sufficient for $\mathcal{A} = C(X)$ and for some other algebras ([10]) and we do not know any complex Banach algebra that would fail such property.

Yet another line of research concerning generalizations of the Gleason-Kahane-Żelazko Theorem was initiated by R. Arens in 1987 ([4]). He asked whether the set $\mathcal{A}^{-1}$ in (1) can be replaced by a smaller set like $\varphi(\mathcal{A})$, for some entire function φ (an analysis of the original proof shows that $\mathcal{A}^{-1}$ can be replaced by $\exp(\mathcal{A})$). After a series of partial results by R. Arens ([4]), C. Badea ([5]), and R. Berntzen and A. Sołtysiak ([6]), the complete answer was obtained recently:

THEOREM 5 ([11]). *Let $\mathcal{A}$ be a complex Banach algebra with a unit e, let φ be a nonconstant entire function, and let F be a linear functional on $\mathcal{A}$. If the function $F \circ \varphi : \mathcal{A} \to \mathbb{C}$ is nonsurjective then $F = 0$, or $F/F(e)$ is multiplicative.*

2.6. Hirschfeld-Żelazko Theorem One of the most important class of Banach algebras are function algebras, also referred as uniform algebras. A function algebra $\mathcal{A}$ is a closed subalgebra of $C(X)$ equipped with the sup norm. Obviously

$$\|f\|^2 = \|f^2\| \quad \text{for } f \in \mathcal{A}, \tag{2}$$

and by the Gelfand Representation Theorem any commutative Banach algebra that satisfies (2) is isometrically isomorphic with a function algebra. Is the assumption that $\mathcal{A}$ is commutative superfluous? Hirschfeld and Żelazko gave an affirmative answer ([1968d]).

THEOREM 6. *Assume A is a complex Banach algebra such that*

$$\|f\|^2 \leq c \|f^2\| \quad \textit{for } f \in \mathcal{A}, \tag{3}$$

for some positive constant c. Then $\mathcal{A}$ is commutative and consequently isomorphic with a function algebra.

This is not only a very elegant result but also one with numerous applications, generalizations and still related open problems. For example - is the result valid for real Banach algebras? Obviously not - we have a very simple counterexample - the noncommutative algebra $\mathbb{H}$ of quaternions. However this is only half of the answer; if a real Banach algebra satisfies (3) then it must be a (possibly noncommutative) function algebra, that is a subalgebra of $C_{\mathbb{H}}(X)$ consisting of continuous $\mathbb{H}$-valued functions on a compact set X ([1]). What if the condition (3) is satisfied on each commutative subalgebra of $\mathcal{A}$ but possibly with different constants c for each subalgebra, must $\mathcal{A}$ be commutative? We do not know.

2.7. Topologizable and nontopologizable algebras Typically in our area of study we investigate properties of a given Banach, or a topological algebra, or of a class of such objects. Why not go deeper and ask how such objects could be created? Given just an algebra, can we always introduce a norm, or at least a topology to make it a topological algebra, preferably a nice one, like a locally convex algebra? Is such a topology unique? Such intriguing questions were considered for decades and Żelazko was attracted by them since eighties while developing formal classification of topological algebras - [1992b] provides a list of interesting problems from that time. In 1990 he showed that the algebra $L(X)$ of all endomorphisms of a vector space X is topologizable as a locally convex topological algebra (with jointly continuous multiplication) if and only if it is topologizable as a Banach algebra and this holds if and only if X is of finite dimension ([1990c]). That result was subsequently generalized by V. Müller ([13]) and other. In the following years Żelazko proved several further results on that subject. He showed that any real or complex countably generated algebra has a complete locally convex topology making it a topological algebra ([1994f]). Such a topology however may not be unique. Indeed, in a joint paper with M. Wojciechowski ([1997a]) they proved that the algebra $P(t)$ of all polynomials of one variable admits a continuum different complete locally convex topologies. It has been known since seventies ([7],[8]) that for a locally convex space X the algebra $B(X)$ of all continuous endomorphisms of X is topologizable only if X is subnormed. In one of the most recent papers ([2002b]) Żelazko showed that when X is sequentially complete this condition is also sufficient and obtained some other conditions equivalent to topologizability of $B(X)$.

2.8. From subalgebras to the algebra If all commutative subalgebras of a Banach algebra $\mathcal{A}$ are isometric with a uniform algebra then $\mathcal{A}$ is commutative and also a uniform algebra. If a linear functional F on $\mathcal{A}$ is multiplicative on any commutative subalgebra, then F is multiplicative. There are several results like these that allow us to make conclusions about the entire algebra based on properties of nice subalgebras. Żelazko was very interested in such and related problems for many years, in fact the two statements just mentioned follow from the Hirshfeld-Żelazko Theorem and from the Gleason-Kahane-Żelazko Theorem, respectively. A closely related question asks when an algebra could be generated by a small number of much simpler subalgebras.

In general, for a topological algebra such questions have often negative answer. For example: there is a semitopological but non-topological algebra (that is an algebra with separately but not jointly continuous multiplication) with every commutative subalgebra topological ([1997a]), also there is a non-locally convex topological algebra with all commutative subalgebras locally convex ([1996e]).

The problem whether an algebra $\mathcal{A}$ can be generated by few commutative subalgebras is particularly interesting for $\mathcal{A} = B(X)$ - the algebra of all continuous endomorphisms of a Banach space X. Żelazko showed that $B(X)$ is algebraically generated by two of its commutative subalgebras $\mathcal{A}_1$ and $\mathcal{A}_2$ of square zero whenever $X = X_1 \oplus \cdots \oplus X_n$, where $X_1, \cdots, X_n$ are mutually isomorphic, but $B(X)$ may not be even topologically generated by such two algebras in general ([1990a]). On the other hand there always exist two subalgebras of square zero which topologically generate $B(X)$ with respect to the strong operator topology ([1988b]). If X is separable $B(X)$ can even be topologically generated in strong operator topology by just two elements of $B(X)$ ([1989a]). Subsequently Żelazko proved that many of these type of conditions are equivalent, for example the following three: (i) X is a square, that is, X decomposes into a direct sum of two closed subspaces $X = X_1 \oplus X_2$ where X_1 and X_2 are isomorphic; (ii) $B(X)$ is algebraically generated by two subalgebras of square zero, one of them being of dimension one ([1988b]), and (iii) $B(X)$ is uniformly generated by two subalgebras of square zero, one of them being of dimension one ([1992a]).

Papers by Wiesław Żelazko

[1957a] *On the divisors of zero of the group algebra.* Fund. Math. 45 (1957), 99–102.

[1957b] with A. Białynicki-Birula, *On the multiplicative linear functionals on the Cartesian product of algebras.* Bull. Acad. Polon. Sci. Cl. III. 5 (1957), 589–593.

[1959] *On a certain class of topological division algebras.* Bull. Acad. Polon. Sci. Sér. Sci. Math. Astr. Phys. 7 (1959), 201–203.

[1960a] *On the locally bounded and m-convex topological algebras.* Studia Math. 19 (1960), 333–356.

[1960b] *A theorem on B_0 division algebras.* Bull. Acad. Polon. Sci. Sér. Sci. Math. Astronom. Phys. 8 (1960), 373–375.

[1961a] *On the algebras L_p of locally compact groups.* Colloq. Math. 8 (1961), 115–120.

[1961b] *A theorem on the discrete groups and algebras L_p.* Colloq. Math. 8 (1961), 205–207.

[1962a] *On the radicals of p-normed algebras.* Studia Math. 21 (1962), 203–206.

[1962b] with B. Mitiagin and S. Rolewicz, *Entire functions in B_0-algebras.* Studia Math. 21 (1962), 291–306.

[1962c] *On the analytic functions in p-normed algebras.* Studia Math. 21 (1962), 345–350.

[1963a] *A note on L_p-algebras.* Colloq. Math. 10 (1963), 53–56.

[1963b] *On decomposition of a commutative p-normed algebra into a direct sum of ideals.* Colloq. Math. 10 (1963), 57–60.

[1963c] *Some remarks on topological algebras.* Studia Math., Ser. Spec., Z1 (1963), 141–149.

[1963d] with S. Rolewicz, *Some problems concerning B_0-algebras.* Tensor (N.S.) 13 (1963), 269–276.

[1965a] *Metric generalizations of Banach algebras.* Dissert. Math. (Rozprawy Mat.) 47 (1965), 70 pp.

[1965b] with M. Rajagopalan, *L_p-conjecture for solvable locally compact groups.* J. Indian Math. Soc. (N.S.) 29 (1965), 87–92.

[1966] *On generalized topological divisors of zero in m-convex locally convex algebras.* Studia Math. 28 (1966), 9–16.

[1967a] *On topological divisors of zero in p-normed algebras without unit.* Colloq. Math. 16 (1967), 231–234.

[1967b] *On generalized topological divisors of zero in real m-convex algebras.* Studia Math. 28 (1967), 241–244.

[1967c] Topological Groups and Algebras, Lect. Notes Math. Inst. Polish Acad. Sc. (1967).

[1968a] with J.-P. Kahane, *A characterization of maximal ideals in commutative Banach algebras.* Studia Math. 29 (1968), 339–343.

[1968b] *A characterization of multiplicative linear functionals in complex Banach algebras.* Studia Math. 30 (1968), 83–85.

[1968c] *Concerning extension of multiplicative linear functionals in Banach algebras.* Studia Math. 31 (1968), 495–499.

[1968d] with R. A. Hirschfeld, *On spectral norm Banach algebras.* Bull. Acad. Polon. Sci. Sér. Sci. Math. Astronom. Phys. 16 (1968), 195–199.

[1968e] *Banach Algebras* (Polish). Polish Sc. Pub. (Państwowe Wydawnictwo Naukowe), Vol. 32, Warsaw, 1968. 178 pp.

[1968f] *Concerning non-commutative Banach algebras of type ES.* Colloq. Math. 20 (1969), 121–126.

[1969] *On m-convex B_0-algebras of type ES.* Colloq. Math. 20 (1969), 299–304.

[1970a] *A characterization of Šilov boundary in function algebras.* Comment. Math. (Prace Mat.) 14 (1970), 59–64.

[1970b] *A characterization of Shilov boundary and non-removable ideals in function algebras.* Proc. Conf. Algebres de Fonctions, Grenoble (1970), 70-73.

[1971a] *A power series with finite domain of convergence.* Comment. Math. (Prace Mat.) 15 (1971), 115-117

[1971b] *On permanently singular elements in commutative m-convex locally convex algebras.* Studia Math. 37 (1971), 181–190.

[1971c] *Selected Topics in Topological Algebras.* Aarhus Univ. Lecture Notes 31 (1971), iii+176 pp.

[1972] *On a certain class of non-removable ideals in Banach algebras.* Studia Math. 44 (1972), 87–92.

[1973a] *Banach Algebras.* Translated from the Polish by Marcin E. Kuczma. Elsevier Publishing Co., Amsterdam-London-New York; PWN–Polish Scientific Publishers, Warsaw, 1973. xi+182 pp.

[1973b] *On a problem concerning joint approximate point spectra.* Studia Math. 45 (1973), 239–240.

[1973c] *On multiplicative linear functionals.* Colloq. Math. 28 (1973), 251–253.

[1974a] *Concerning a problem of Arens on removable ideals in Banach algebras.* Colloq. Math. 30 (1974), 127–131.

[1974b] with Z. Słodkowski, *On joint spectra of commuting families of operators.* Studia Math. 50 (1974), 127–148.

[1975a] *On the Burnside problem for locally compact groups.* Symposia Mathematica, Vol. XVI (1975), 409–416.

[1975b] *Topological Algebras.* Lecture Notes, McMaster Univ. 1975.

[1976a] *On maximal ideals in commutative m-convex algebras.* Studia Math. 58 (1976), 291–298.

[1978a] *Concerning algebras of functions with restricted growth.* Comment. Math. Special Issue 1 (1978), 381–384.

[1978b] *Concerning nonremovable ideals in commutative m-convex algebra*s. Demonstratio Math. 11 (1978), 239–245.

[1978c] *On some classes of ideals in commutative Banach algebras.* Rend. Sem. Mat. Fis. Milano 48 (1978), 51–58 (1980).

[1979a] *An axiomatic approach to joint spectra. I.* Studia Math. 64 (1979), 249–261.

[1979b] *Some open problems in topological algebras.* Proc. Int. Conf. Leipzig 1977 (1979), 381-384.

[1980] *A characterization of LC-nonremovable ideals in commutative Banach algebras.* Pacific J. Math. 87 (1980), 241–248.

[1981a] *A domination theorem for function algebras.* Colloq. Math. 44 (1981),, 317–321.

[1981b] *On domination and separation of ideals in commutative Banach algebras.* Studia Math. 71 (1981), 179–189.

[1982] with Z. Słodkowski, *A note on semicharacters.* Spectral theory (Warsaw, 1977), 397–402, Banach Center Publ., 8, PWN, Warsaw, 1982.

[1982b] Editor: *Spectral theory. Papers presented during the semester held at the Stefan Banach International Mathematical Center, Warsaw, September 23–December 16, 1977.* Banach Center Publications, 8. PWN—Polish Scientific Publishers, Warsaw, 1982. 603 pp.

[1983] *On permanent radicals in commutative locally convex algebras.* Studia Math. 75 (1983), 265–272.

[1984a] *On Ideal Theory in Banach and Topological Algebras.* Monografías del Instituto de Matemáticas [Monographs of the Institute of Mathematics], 15. Universidad Nacional Autónoma de México, México, 1984. iii+152 pp.

[1984b] *Concerning characterization of permanently singular elements in commutative locally convex algebras.* Mathematical structure—computational mathematics—mathematical modelling, 2, 326–333, Publ. House Bulgar. Acad. Sci., Sofia, 1984.

[1984c] *On nonremovable ideals in commutative locally convex algebras.* Studia Math. 77 (1984), 133–154.

[1984d] *Two problems concerning separation of ideals in group algebras.* in Linear and Complex Analysis Problem Book. Lect. Notes in Math. 1043 (1984), 68-69.

[1985a] *A non-m-convex algebra on which operate all entire functions.* Ann. Polon. Math. 46 (1985), 389–394.

[1985b] with H. Arizmendi Peimbert, *A B_0-algebra without generalized topological divisors of zero.* Studia Math. 82 (1985), 191–198.

[1986] *Topological divisors of zero, their applications and generalization.* Geometry seminars, 1985 (Italian) (Bologna, 1985), 175–191, Univ. Stud. Bologna, Bologna, 1986

[1987a] *Functional continuity of commutative m-convex B_0-algebras with countable maximal ideal spaces.* Colloq. Math. 51 (1987), 395–399.

[1987b] *Concerning locally convex algebras without generalized topological divisors of zeros.* Funct. Approx. Comment. Math. 17 (1987), 11–20.

[1987c] *On generalized topological divisors of zero.* Studia Math. 85 (1987), 137–148.

[1988a] *On generalizations of a Shilov theorem.* Rend. Sem. Mat. Fis. Milano 55 (1985), 139–145 (1988).

[1988b] *Algebraic generation of $B(X)$ by two subalgebras with square zero.* Studia Math. 90 (1988), 205–212.

[1988c] *$B(X)$ is generated in strong operator topology by two subalgebras with square zero.* Proc. Roy. Irish Acad. Sect. A 88 (1988), 19–21.

[1988d] *O pewnym problemie typu Burnside'a* (Polish), Delta 8n (176), 188, 1-2.

[1989a] with V. Müller, *$B(X)$ is generated in strong operator topology by two of its elements.* Czechoslovak Math. J. 39 (114) (1989), no. 3, 486–489.

[1989b] Co-Editor (with A. Hulanicki and P. Wojtaszczyk): *Selected papers of Antoni Zygmund. Vol. 1.* Mathematics and its Applications (East European Series), 41/1. Kluwer Academic Publishers Group, Dordrecht, 1989. lxii+443 pp.

[1989c] Co-Editor (with A. Hulanicki and P. Wojtaszczyk): *Selected papers of Antoni Zygmund. Vol. 2.* Mathematics and its Applications (East European Series), 41/2. Kluwer Academic Publishers Group, Dordrecht, 1989. xxii+444 pp.

[1989d] Co-Editor (with A. Hulanicki and P. Wojtaszczyk): *Selected papers of Antoni Zygmund. Vol. 3.* Mathematics and its Applications (East European Series), 41/3. Kluwer Academic Publishers Group, Dordrecht, 1989. xxii+457 pp.

[1990a] *$B(H)$ is generated by two of its commutative subalgebras,* Proc. Invariant Subspaces and Allied Topics (1986), Norosa Publ. House 1990, 144-146.

[1990b] *A density theorem for F-spaces.* Studia Math. 96 (1990), 159–166.

[1990c] *Example of an algebra which is nontopologizable as a locally convex topological algebra.* Proc. Amer. Math. Soc. 110 (1990), 947–949.

[1990d] *Władyslaw Orlicz* (Polish). Rocznik Tow. Nauk. Warszawskiego 53 (1990), 76-77.

[1991] *Concerning trivial maximal abelian subalgebras of $B(X)$.* Pliska Stud. Math. Bulgar. 11 (1991), 113–116.

[1991b] *On a problem of Fell and Doran.* Colloq. Math. 62 (1991), 31–37

[1991c] *Some open problems and recent results in topological algebras.* in Functional Analysis and Related Topics, World Sc. 1991, 58-64.

[1992a] *Concerning generation of $B(X)$ by two subalgebras of square zero.* Funct. Approx. Comment. Math. 20 (1992), 45–49

[1992b] *On certain open problems in topological algebras.* Rend. Sem. Mat. Fis. Milano 59 (1989), 49–58 (1992).

[1992c] *Extending seminorms in locally pseudoconvex algebras.* Functional analysis and operator theory (New Delhi, 1990), 215–223, Lecture Notes in Math., 1511, Springer, Berlin, 1992.

[1992d] with M. Chō, *On geometric spectral radius of commuting n-tuples of operators.* Hokkaido Math. J. 21 (1992), 251–258.

[1992e] *A subfield of a complex Banach algebra is not necessarily topologically isomorphic to a subfield of C.* Colloq. Math. 63 (1992), 135–137.

[1992f] *Stefan Banach (1992-1945).* Europ. Math. Soc. Newsletter (5) (1992), p. 23.

[1994a] *Four problems concerning joint spectra,* in in Linear and Complex Analysis Problem Book 3. Lect. Notes in Math. 1573 (1994), 105-106.

[1994b] *On strongly closed subalgebras of* $B(X)$. Colloq. Math. 67 (1994), no. 2, 289–295.

[1994c] *Concerning entire functions in* B_0*-algebras.* Studia Math. 110 (1994), 283–290.

[1994d] *Generation of* $B(X)$ *by two commutative subalgebras - results and open problems.* Functional analysis and operator theory (Warsaw, 1992), 363–367, Banach Center Publ., 30, Polish Acad. Sci., Warsaw, 1994.

[1994e] Open problems and some results on strongly closed subalgebras of $B(X)$. Acta Comm. Univ. Tartuensis 970 (1994), 125-130.

[1994f] *On topologization of countably generated algebras.* Studia Math. 112 (1994), 83–88.

[1994g] *What is known and what is not known about multiplicative linear functionals.* Topological vector spaces, algebras and related areas (Hamilton, ON, 1994), 102–115, Pitman Res. Notes Math. Ser., 316, Longman Sci. Tech., Harlow, 1994.

[1994h] *Further examples of locally convex algebras.* Topological vector spaces, algebras and related areas (Hamilton, ON, 1994), 162–171, Pitman Res. Notes Math. Ser., 316, Longman Sci. Tech., Harlow, 1994.

[1995] with A. Kokk, *On vector spaces and algebras with maximal locally pseudoconvex topologies*. Studia Math. 112 (1995), 195–201.

[1996a] *Concerning topologization of* $P(t)$. Acta Univ. Lodz. Folia Math. No. 8 (1996), 153–159.

[1996b] *The strongest vector space topology is locally convex on separable linear subspaces.* Ann. Polon. Math. 66 (1997), 275–282.

[1996c] *Concerning topologization of real or complex algebras.* Colloq. Math. 71 (1996), no. 1, 111–113.

[1996d] *A non-Banach m-convex algebra all of whose closed commutative subalgebras are Banach algebras.* Studia Math. 119 (1996), 195–198.

[1996e] *A non-locally convex topological algebra with all commutative subalgebras locally convex*. Studia Math. 120 (1996), 89–94.

[1996f] *A locally bounded algebra all of whose closed commutative subalgebras are Banach algebras.* Comment. Math. (Prace Mat.) 36 (1996), 265–270.

[1997a] with M. Wojciechowski, *Non-uniqueness of topology for algebras of polynomials.* Colloq. Math. 72 (1997), 111–121.

[1997b] *A semitopological algebra without proper closed subalgebras.* Colloq. Math. 74 (1997), 239–242.

[1997c] *Frechet algebra.* in Encyclopedia of Math., Supplement, Vol. 1, Kluver Acad. Pub. (1997), 257-258.

[1997d] *Frechet topology.* in Encyclopedia of Math., Supplement, Vol. 1, Kluver Acad. Pub. (1997), 258-259.

[1997e] *Mazur-Orlicz theorem.* in Encyclopedia of Math., Supplement, Vol. 1, Kluver Acad. Pub. (1997), 369-370.

[1997f] *Topological simplicity of a certain LF-algebra.* Period. Math. Hungar. 35 (1997), 145–148.

[1998a] *Topologization of algebras - the results and open problems.* in Functional Analysis, Narosa Pub. House, 1998.

[1998b] *On some old problems and new questions in topological algebras.* Functional analysis with current applications in science, technology and industry (Aligarh, 1996), 3–19, Pitman Res. Notes Math. Ser., 377, Longman, Harlow, 1998.

[1998c] *A density theorem for algebra representations on the space* (s). Studia Math. 130 (1998), 293–296.

[1999] *On a problem of Fell and Doran.* Operator inequalities and related topics (Japanese) (Kyoto, 1998). Sūrikaisekikenkyūsho Kōkyūroku No. 1080 (1999), 137–139.

[2000a] with M. Chō, R. E. Curto, and T. Huruya, *Cartesian form of Putnam's inequality for doubly commuting hyponormal* n*-tuples.* Indiana Univ. Math. J. 49 (2000), 1437–1448.

[2000b] *A characterization of commutative Fréchet algebras with all ideals closed.* Studia Math. 138 (2000), 293–300.

[2000c] *On* m*-convexity of commutative real Waelbroeck algebras.* Comment. Math. (Prace Mat.) 40 (2000), 235–240.

[2001a] *Characterizations of* Q*-algebras of type* F *and of* F*-algebras with all ideals closed.* General topological algebras (Tartu, 1999), 183–189, Math. Stud. (Tartu), 1, Est. Math. Soc., Tartu, 2001.

[2001b] *A generalization of multiplicatively absorbing algebras.* General topological algebras (Tartu, 1999), 190–194, Math. Stud. (Tartu), 1, Est. Math. Soc., Tartu, 2001.

[2001c] *Open problems and some results on strongly closed subalgebras of* $B(X)$. Tartu Ül. Toimetised No. 970 (1994), 125–130.

[2002a] *Concerning closed invariant subspaces for endomorphisms of the space* (s). Period. Math. Hungar. 44 (2002), no. 2, 239–242.

[2002b] *When is* $L(X)$ *topologizable as a topological algebra?* Studia Math. 150 (2002), 295-303.

[2002c] F*-algebras: some results and open problems.* Proc. Int. Conf. Funct. Anal. Appl. Lviv, (2002), Elsevier.

[2003a] *Concerning complexification of* Q*-algebras.* Proc. ICTAA 3.

[2003b] *Concerning topologization of the algebra* $L(X)$. Proc. ICAA 3.

[2003c] *When a commutative* F*-algebra has a dense principal ideal.* Proc. ICTAA 4.

References

[1] M. Abel and K. Jarosz, *Noncommutative Uniform Algebras*, preprint.

[2] B. Aupetit, *Sur les transformations qui conservent le spectre.* Banach algebras '97 (Blaubeuren), 55–78, de Gruyter, Berlin, 1998.

[3] B. Aupetit, *Spectrum-preserving linear mappings between Banach algebras or Jordan-Banach algebras.* J. London Math. Soc. (2) 62 (2000), no. 3, 917–924.

[4] R. Arens, *On a theorem of Gleason, Kahane and Żelazko.* Studia Math. 87 (1987), no. 2, 193–196.

[5] C. Badea, *The Gleason-Kahane-Żelazko theorem.* Proceedings of the Second International Conference in Functional Analysis and Approximation Theory (Acquafredda di Maratea, 1992). Rend. Circ. Mat. Palermo (2) Suppl. No. 33, (1993), 177–188.

[6] R. Berntzen and A. Sołtysiak, *On a conjecture of Jarosz.* Comment. Math. (Prace Mat.) 36 (1996), 39–45.

[7] J. Esterle, *Sur la non normabilité de certaines algèbres d'opérateurs.* (French) C. R. Acad. Sci. Paris Sér. A 278 (1974), 1037–1040.

[8] J. Esterle, *Sur la métrisabilité de certaines algèbres d'opérateurs.* (French) Rev. Roumaine Math. Pures Appl. 24 (1979), no. 8, 1157–1164.

[9] A. M. Gleason, *A characterization of maximal ideals.* J. Analyse Math. 19 (1967), 171–172.

[10] K. Jarosz, *Finite codimensional ideals in function algebras,* Trans. Amer. Math. Soc. 287 (1985), 779-785.

[11] K. Jarosz, *Multiplicative functionals and entire functions. II.* Studia Math. 124 (1997), no. 2, 193–198.

[12] K. Jarosz, *When is a linear functional multiplicative?* Function spaces (Edwardsville, IL, 1998), 201–210, Contemp. Math., 232, Amer. Math. Soc., Providence, RI, 1999.

[13] V. Müller, *On topologizable algebras*, Studia Math. 99 (1991), 149-153.