Manuel MAESTRE (maestre@uv.es), Faculty of Mathematics, University of Valencia, 46100 Burjasot - Valencia, Spain, ***On Bohr's power series theorem.***

ABSTRACT. In 1914 Harald Bohr published the following surprising result: Suppose that we have $\left|\sum_{k=0}^{\infty} a_k z^k\right| \leq 1$ for each complex number z in the open unit disk. Then $\sum_{k=0}^{\infty} \left|a_k z^k\right| \leq 1$ when $|z| < \frac{1}{3}$, and moreover the radius $\frac{1}{3}$, is best possible. Recently several authors studied Bohr's power series theorem in higher dimension: Given a Banach space $X = (\mathbb{C}^n, \|.\|)$, what is the largest radius $K(B_X)$ (called Bohr radius of the open unit ball of X) such that if $|\sum_{\alpha}^{\infty} a_\alpha z^\alpha| \leq 1$ for all $\|z\| < 1$, then $\sum_{\alpha}^{\infty} |a_\alpha z^\alpha| \leq 1$ when $\|z\| < K(B_X)$? A result of Dineen-Timoney, and Boas-Khavinson states that the scalar sequence $(K(B_{\ell_\infty^n}))$ tends to zero, and that its decay is essentially like $\frac{1}{\sqrt{n}}$. We link this cycle of ideas around multi-variable powers series with local Banach space theory, in particular with our recent research on unconditionality in spaces of m-homogeneous polynomials on X. Estimates for $K(B_X)$ are then obtained by probabilistic methods. The main probabilistic tool we obtain and apply is an estimate for the expectation of the norm of Gaussian random polynomials on Banach spaces. This is a polynomial counterpart of an important inequality due to Chevet.