



Geoff DIESTEL (geoff_diestel@hotmail.com),
Mathematics Department, 202 Mathematical Sciences
Building, University of Missouri, Columbia, MO 65211,
USA *Sobolev Spaces with Trivial Isometries.*

ABSTRACT. For a bounded domain $E \subset \mathbb{R}^n$, $p > 0$
and $k \in \mathbb{N}$ we consider the Sobolev space $W_p^k(E)$ of k -
times continuously differentiable real-valued functions
 f on E equipped with the norm

$$\|f\|^p = \|f\|_{L_p(E)}^p + \sum_{1 \leq |\alpha| \leq k} \|D^\alpha f\|_{L_p(E)}^p,$$

where $D^\alpha f = \frac{\partial^{|\alpha|} f}{\partial x_1^{\alpha_1} \dots \partial x_n^{\alpha_n}}$ and $|\alpha| = \sum_{j=1}^n \alpha_j$ for multiindices $\alpha = (\alpha_1, \dots, \alpha_n)$
and $\alpha_j \in \mathbb{N} \cup \{0\}$ for $1 \leq j \leq n$. In this note we show that if p is not an even
integer then the structure of linear (surjective) isometries of the spaces $W_p^k(E)$
strongly depends on the geometry of the domain E . We prove the following. Let
 E and G be closed bounded connected domains in \mathbb{R}^n such that $m(E \setminus \text{int}(E)) =$
 0 . Let $p > 0$, $p \notin 2\mathbb{N}$ and $k \in \mathbb{N}$. Then the space $W_p^k(E)$ is isometric to $W_p^k(G)$ if
and only if the domains E and G coincide up to the composition of a translation
and a sign-changing permutation of coordinates.