Eggert BRIEM (briem@rhi.hi.is), Department of Mathematics, University of Iceland, 3 Dunhaga, 107 Reykjavik, Iceland, ***An extension of a theorem of Wermer, Bernard, Sidney and Hatori to algebras of functions on locally compact spaces.***

ABSTRACT. Let X be a compact Hausdorff space and B a uniformly closed subspace of $C(X, \mathbf{R})$ which separates the points of X and contains the constant functions. A version of the Stone-Weierstrass theorem says that if $b^2 \in B$ for all $b \in B$ then $B = C(X, \mathbf{R})$. This result does clearly not hold if, instead of assuming that B is uniformly closed, one assumes that B is a Banach space in some norm which dominates the sup-norm as the example of any non-trivial real Banach function algebra shows. However, if B is the real part of a uniform algebra, a theorem of J. Wermer says that if $b^2 \in B$ for all $b \in B$ then $B = C(X, \mathbf{R})$. Let us say that a real-valued function h, defined on an interval I of the real line, *operates* on B if $h \circ b \in B$ whenever $b \in B$ and b maps X into I. Thus, $b^2 \in B$ for all $b \in B$ means that $h(t) = t^2$ operates on B. The Stone-Weierstrass theorem was generalized by K. deLeeuw and Y. Katznelson, they showed that $h(t) = t^2$ can be replaced by any continuous non-affine function and the theorem of J. Wermer was similarily generalized by A. Bernard, S. Sidney and O. Hatori; here one can do without the continuity assumption, a function operating on the real part of a uniform algebra is automatically continuous. In case X is locally compact and B is a subspace of $C_0(X, \mathbf{R})$, the functional calculus for B may be non-trivial. What then about the real part of a uniformly closed subalgebra of $C_0(X, \mathbf{C})$? Wermer's theorem clearly extends to this situation. It turns out that the functional calculus is trivial for the locally compact case. To prove this one has to do more than just adapt the proofs for the compact case to the locally compact situation. The proofs in the compact case use the precence of the constant functions, especially the proof of continuity of the operating function and the proof of density of B in $C(X, \mathbf{R})$.