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Armando R. Villena, Departamento de Análisis Matemático, Universidad de Granada, Granada, Spain; (avillena@ugr.es)

Operators shrinking the Arveson spectrum.

ABSTRACT. Let G be a locally compact abelian group and let τ_X and τ_Y be strongly continuous representations of G on Banach spaces X and Y, respectively. If $\Phi\colon X \to Y$ is a bounded linear operator intertwining τ_X and τ_Y, then it is immediate that $\mathrm{sp}_{\tau_X}(\Phi x) \subset \mathrm{sp}_{\tau_Y}(x)$ for each $x \in X$. Here $\mathrm{sp}_\tau(z)$ stands for the local Arveson spectrum of the representation τ at the point z. We will discuss whether the converse holds true. As an application we show that if S and T are invertible bounded linear operators on a complex Banach space X which have polynomial growth, i.e. $\sup_{k\neq 0}|k|^{-\alpha}\|S^k\|,\ \sup_{k\neq 0}|k|^{-\alpha}\|T^k\| < \infty$ for some $\alpha \geq 0$, and if the local spectrum of S at every $x \in X$ is contained in the corresponding local spectrum of T, then the following identity holds

$$\sum_{k=0}^{N} \binom{N}{k} (-1)^k S^{N-k} T^k = 0$$

for each integer N with $N \geq 2\alpha$.