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Operator algebras associated with subproduct systems .

ABSTRACT. A product system of Hilbert spaces over a semigroup S is a family $\{H(s) : s \in S\}$ such that, for every $s, t \in S$, there is an isomorphism $U_{s,t} : H(s) \otimes H(t) \to H(s+t)$ and these maps "compose associatively". Given such a system one can define shift operators and can consider the Banach algebra (or the C^*-algebra) that they generate. These algebras have been studied in the literature. This was also generalized to the case where the Hilbert spaces where replaced by C^*-correspondences (or C^*-Hilbert bimodules). Product systems of of C^*-correspondences also play an important role in the dilation theory of a semigroup of completely positive maps.

Examining the role of product systems in the dilation theory brings one to the conclusion that the natural object of study should be *subproduct systems*. These are defined in a similar way except that the maps $U_{s,t}$ are assumed to be coisometries and not necessarily unitaries.

In this work, which is joint with Orr Shalit, we start the systematic study of these subproduct systems and the operator algebras generated by "shifts" on these systems. This generalizes the work of Arveson on (commuting) d-shifts and the study (by several researchers) of algebras generated by q-commuting tuples. It also generalizes the study of Matsumoto's subshift algebras and is related to some work of G. Popescu.