

Banach Algebras 2009

*A conference supported by the European Science Foundation under the
ESF-EMS-ERC COM partnership*

*July 14-24, 2009, Stefan Banach International Mathematical Center,
Będlewo, Poland*

Volker Runde, Department of Mathematical and Statistical Sciences,
University of Alberta, Edmonton, AB, Canada T6G 2G1 (vrunde@ualberta.ca)

Amenability of $B(\ell^p)$ for $p \neq 2$.

ABSTRACT. The late Barry Johnson asked whether $B(E)$, the Banach algebra of all bounded linear operators on a Banach space E , could be amenable for any infinite-dimensional E . For many years, this question was open even for $E = \ell^p$ with $p \neq 2$. Only recently, the case of $p = 1, \infty$ was settled through work by C. J. Read, G. Pisier, and N. Ozawa.

In this talk, we explore consequences of the hypothetical amenability of $B(\ell^p)$, and, in particular, link it to the notion of ultra-amenability as recently introduced by Matt Daws. (A Banach algebra is ultra-amenable if each of its ultrapowers is amenable.) If $B(\ell^p)$ is amenable for some $p \in (1, \infty) \setminus \{2\}$, then $\ell^\infty(I, K(E))$ is amenable for any index set I and any L^p -space E in the sense of Lindenstrauss–Pełczyński. In particular, $\ell^\infty(I, K(\ell^p \oplus \ell^2))$ is amenable, so that $K(\ell^p \oplus \ell^2)$ is ultra-amenable; this is remarkable because $B(\ell^p \oplus \ell^2)$ is known to be non-amenable.

This is joint work with Matt Daws.