

Banach Algebras 2009

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Homological dimensions of Köthe algebras

ABSTRACT. Köthe spaces consist of complex sequences with prescribed growth conditions at infinity. To be more precise, given a countable set P of nonnegative sequences satisfying some natural conditions, the *Köthe space* $\lambda(P)$ is defined by

$$\lambda(P) = \left\{ x = (x_i) \in \mathbb{C}^{\mathbb{N}} : \|x\|_p = \sum_i |x_i| p_i < \infty \quad \forall p \in P \right\}.$$

Clearly, every Köthe space is a Fréchet space w.r.t. the topology determined by the seminorms $\{\|\cdot\|_p : p \in P\}$. Many natural Köthe spaces are in fact Fréchet algebras under pointwise multiplication (for example, this is the case provided that $p_i \geq 1$ for all $p \in P$, $i \in \mathbb{N}$).

We give a survey of recent results on homological properties of Köthe algebras, with an emphasis on biprojectivity, biflatness, and homological dimension. Our main results are explicit formulas for the global dimension, the weak global dimension, the bidimension, and the weak bidimension of $\lambda(P)$ given in terms of P . We show, in particular, that the only possible values for the above dimensions of $\lambda(P)$ are 0, 1, 2, and ∞ . Some related results on the approximate contractibility of $\lambda(P)$ will also be presented.