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***Functions of perturbed operators.***

ABSTRACT. I am going to speak about recent joint results with A.B. Aleksandrov. It is well known that a Lipschitz function does not have to be operator Lipschitz. In other words, the inequality  $|f(x) - f(y)| \leq \text{const} |x - y|$  does not imply that  $\|f(A) - f(B)\| \leq \text{const} \|A - B\|$  for self-adjoint operators  $A$  and  $B$ . It turned out that the situation dramatically changes if we consider functions in Hölder–Zygmund classes. We prove that if  $0 < \alpha < 1$  and  $f$  is in the Hölder class  $\Lambda_\alpha(\mathbb{R})$ , then for arbitrary self-adjoint operators  $A$  and  $B$  with bounded  $A - B$ , the operator  $f(A) - f(B)$  is bounded and  $\|f(A) - f(B)\| \leq \text{const} \|A - B\|^\alpha$ . We prove a similar result for functions  $f$  of Zygmund class  $\Lambda_1(\mathbb{R})$ :  $\|f(A+K) - 2f(A) + f(A-K)\| \leq \text{const} \|K\|$ , where  $A$  and  $K$  are self-adjoint operators. Similar results also hold for all Hölder–Zygmund classes  $\Lambda_\alpha(\mathbb{R})$ ,  $\alpha > 0$ . We also study properties of the operators  $f(A) - f(B)$  for  $f \in \Lambda_\alpha(\mathbb{R})$  and self-adjoint operators  $A$  and  $B$  such that  $A - B$  belongs to the Schatten–von Neumann class  $\mathbf{S}_p$ . We consider the same problem for higher order differences.