

Banach Algebras 2009

*A conference supported by the European Science Foundation under the
ESF-EMS-ERCOT partnership*

*July 14-24, 2009, Stefan Banach International Mathematical Center, Bedlewo,
Poland*

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On the reflexivity deficit.

ABSTRACT. Let X, Y be complex vector spaces and let $\mathcal{A} \subset \mathcal{L}(X, Y)$ be a
subspace. The reflexive hull of \mathcal{A} is the set of all operators $T : X \rightarrow Y$ satisfying
 $Tx \in \mathcal{A}x := \{Ax : A \in \mathcal{A}\}$ for all $x \in X$.

We show that if $\dim \mathcal{A} = n < \infty$ then the reflexive hull of \mathcal{A} is at most
 $\frac{n(n+1)}{2}$ dimensional; the estimate is optimal.

The main tool is the following result which is a generalization of the non-
emptiness of the spectrum of a square matrix (for $k = 2$ and $m = n$): if
 $\mathcal{A} \subset M_{m \times n}$ is a k -dimensional subspace with $k \leq n + 1$ then

$$\dim \bigcap_{A \in \mathcal{A}, A \neq 0} \operatorname{Im} A \leq n - k + 1.$$

This is a joint work with C. Ambrozie and B. Kuzma.