

Banach Algebras 2009

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Non-Hausdorff symmetries of C^* -algebras-algebras.

ABSTRACT. Badly behaved quotient spaces are commonly described by groupoids and their convolution algebras. The symmetry groups of such quotient spaces may be non-Hausdorff as well. Higher category theory provides generalisations of groups and group actions that are appropriate to study such non-Hausdorff symmetries of algebras and C^* -algebras. A 2-category has three levels of data: objects, arrows between objects, and arrows between arrows. The first example is the 2-category of categories, which comprises categories, functors between categories, and natural transformations between functors. We enrich the category of C^* -algebras by viewing unitary intertwiners between $*$ -homomorphisms as arrows between morphisms. This 2-category allows us to formalise in which sense inner automorphisms are trivial, and it provides a good notion of a group action that is defined up to inner automorphisms. The subtle point is that we must require the unitaries that implement inner automorphisms to satisfy a certain cocycle condition. Such twisted group actions were already studied by Busby and Smith. General 2-category theory explains the cocycle conditions that appear for twisted actions and for outer equivalence of actions. Non-Hausdorff symmetry groups of C^* -algebras manifest themselves by actions of 2-categories on C^* -algebras. I will discuss some examples of this such as the gauge symmetry of rotation algebras.