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The higher-dimensional amenability of tensor products of Banach algebras

ABSTRACT. We investigate the higher-dimensional amenability of tensor products $\mathcal{A}\widehat{\otimes}\mathcal{B}$ of Banach algebras $\mathcal{A}$ and $\mathcal{B}$. Recall that, for any $n \geq 1$, a Banach algebra $\mathcal{A}$ is called *n-amenable* if the continuous Hochschild cohomology $\mathcal{H}^n(\mathcal{A}, X^*) = \{0\}$ for every Banach $\mathcal{A}$-bimodule X. It is clear that $\mathcal{A}$ is n-amenable but not $(n-1)$-amenable if and only if the *weak bidimension* of $\mathcal{A}$

$$db_w\mathcal{A} = \inf \ \{n : \mathcal{H}^{n+1}(\mathcal{A}, X^*) = \{0\} \text{ for all Banach } \mathcal{A}\text{−bimodule } X\}$$

is equal to $(n-1)$. In 1996 Yu. Selivanov remarked without proof that the weak bidimension db_w of the tensor product $\mathcal{A}\widehat{\otimes}\mathcal{B}$ of Banach algebras $\mathcal{A}$ and $\mathcal{B}$ with bounded approximate identities (b.a.i.) satisfies

$$db_w\mathcal{A}\widehat{\otimes}\mathcal{B} = db_w\mathcal{A} + db_w\mathcal{B}.$$

In 2002 he gave a proof of the formula in the particular case of algebras with *identities*, and his proof depends heavily on the existence of identities. In this talk we show that the formula is correct for algebras with b.a.i. We prove further that the formula does *not* hold for algebras with no b.a.i, nor for algebras with only 1-sided b.a.i. The well-known trick of adjoining an identity to the algebra does not work for the tensor product of algebras. The homological properties of the tensor product algebras $\mathcal{A}\widehat{\otimes}\mathcal{B}$ and $\mathcal{A}_+\widehat{\otimes}\mathcal{B}_+$ are different; here $\mathcal{A}_+$ is the Banach algebra obtained by adjoining an identity to $\mathcal{A}$. For example, for biflat Banach algebras $\mathcal{A}$ and $\mathcal{B}$ which have left or right, but not two-sided, b.a.i., we have $db_w\mathcal{A}\widehat{\otimes}\mathcal{B} \leq 1$ and $db_w\mathcal{A}_+\widehat{\otimes}\mathcal{B}_+ = db_w\mathcal{A} + db_w\mathcal{B} = 2$.