

Banach Algebras 2009

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Approximation properties of Banach spaces and pointwise approximability of operators.

ABSTRACT. Let A be a linear subspace in the space $\mathcal{L}(X)$ of all bounded linear operators on a Banach space X . The space X has the A -approximation property if, for every compact set $K \subset X$ and every $\varepsilon > 0$, there exists an operator $S \in A$ such that $\|Sx - x\| < \varepsilon$, for all $x \in K$. Substituting A with the space of all finite rank operators $\mathcal{F}(X)$ and compact operators $\mathcal{K}(X)$ will result in classical notions of approximation property and compact approximation property, respectively.

It is a fundamental result that the approximation property can be described in terms of approximability of compact operators by finite-rank operators.

We shall discuss possible extensions of this result to more general contexts as above. In particular, given an operator ideal \mathcal{B} , such that $\mathcal{K} \subset \mathcal{B} \subset \mathcal{W}$ and $A \circ \mathcal{B}^{dual} \subset \mathcal{K}$, (here $\mathcal{B}^{dual} = \{T \in \mathcal{L} : T^* \in \mathcal{B}\}$); we shall characterize the A -approximation property in terms of pointwise approximability of operators T belonging to \mathcal{B} by operators of the form ST , where $S \in A$. The approximation will also preserve a certain metric condition, i.e. $\|ST\| \leq \|T\|$.

Among others, this characterization will enable us to present a number of known results on compact approximation property as applications.