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Gleason parts properties for positive measures.

ABSTRACT. Let $\mathcal{A}$ be a function algebra on a compact set X. Denote by $P(X)$ the set of all probability Borel measures on X. We say that $\mu \in P(X)$ belongs to the same Gleason part as $\eta \in P(X)$ if there is $c > 0$ such that

$$\frac{1}{c} < \frac{\int u\, d\mu}{\int u\, d\eta} < c, \quad u \in \operatorname{Re} \mathcal{A}, u > 0.$$

The above equivalence relation is an extension of the well known relation defined for linear-multiplicative functionals on $\mathcal{A}$ and has the following properties which partially generalize analogous properties for linear-multiplicative functionals.

Theorem. *Denote by $\mathcal{A}_\phi$ the kernel of $\phi \in \mathcal{M}$, and formulate the following statements for $\phi \in \mathcal{M}$, $\mu \in P(X)$.*

1. *ϕ and μ are in the same part of $P(X)$.*
2. *$\|\phi - \mu\| < 2$, the norm of the dual of $\mathcal{A}$.*
3. *The norm of the restriction of μ to $\mathcal{A}_\phi$ is less than one.*
4. *Whenever $\{f_n\}_{n=1}^{\infty}$ is a sequence in $\mathcal{A}$ such that $\|f_n\| \leqslant 1$ and $|\int f_n\, d\mu| \to 1$, then $|f_n(\phi)| \to 1$.*

Then (1) $\implies$ (2) $\iff$ (3) $\iff$ (4)