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Compact and weakly compact derivations from commutative Banach algebras

ABSTRACT. We consider when a derivation from a commutative Banach algebra into its dual is a compact or weakly compact linear map. Before turning our attention to some specific cases, we show that if there are no compact derivations from a commutative Banach algebra A to its dual then there are no compact derivations from A to any symmetric A -bimodule and that the analogous result holds for weak compactness. We then characterize when derivations from the convolution algebra $\ell^1(Z_+)$ to its dual are compact and when they are weakly compact. The case of the compact derivations is rather simple and the space of all such derivations is easily identified with c_0 . The characterisation of the weakly compact derivations is combinatorial, in terms of “translation-finite” subsets of Z_+ . This last characterisation comes from joint work with Yemon Choi. Finally, we contrast $\ell^1(Z_+)$ with the disc algebra $A(\Delta)$, from which all bounded derivations into the dual are compact. This follows from a complete characterisation of the bounded derivations from $A(\Delta)$ to $A(\Delta)$ in a separate piece of joint work with Yemon Choi.