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J. F. Feinstein, School of Mathematical Sciences, University of Nottingham, Nottingham NG7 2RD, UK (Joel.Feinstein@nottingham.ac.uk)

Normed algebras of differentiable functions on compact plane sets

ABSTRACT. We investigate the completeness and completions of the normed algebras $(D^{(1)}(X), \|\cdot\|)$ for perfect, compact plane sets X. In particular, we construct a radially self-absorbing, compact plane set X such that the normed algebra $(D^{(1)}(X), \|\cdot\|)$ is not complete. This solves a question of Bland and Feinstein. We also prove that there are several classes of connected, compact plane sets X for which the completeness of $(D^{(1)}(X), \|\cdot\|)$ is equivalent to the pointwise regularity of X. For example, this is true for all rectifiably connected, polynomially convex, compact plane sets with empty interior, for all star-shaped, compact plane sets, and for all Jordan arcs in $\mathbb{C}$.

In an earlier paper of Bland and Feinstein, the notion of an $\mathcal{F}$-derivative of a function was introduced, where $\mathcal{F}$ is a suitable set of rectifiable paths, and with it a new family of Banach algebras $D^{(1)}_{\mathcal{F}}(X)$ corresponding to the normed algebras $D^{(1)}(X)$. We obtain stronger results concerning the questions when $D^{(1)}(X)$ and $D^{(1)}_{\mathcal{F}}(X)$ are equal, and when the former is dense in the latter. In particular, we show that equality holds whenever X is '$\mathcal{F}$-regular'.

An example of Bishop shows that the completion of $(D^{(1)}(X), \|\cdot\|)$ need not be semisimple. We show that the completion of $(D^{(1)}(X), \|\cdot\|)$ is semisimple whenever the union of all the rectifiable Jordan arcs in X is dense in X.

We prove that the character space of $D^{(1)}(X)$ is equal to X for all perfect, compact plane sets X, whether or not $(D^{(1)}(X), \|\cdot\|)$ is complete. In particular, characters on the normed algebras $(D^{(1)}(X), \|\cdot\|)$ are automatically continuous.

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