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Quantum duality and noncommutative Arens-Mackey theorem.

ABSTRACT. In the present talk we discuss the duality theory for quantum (or local operator) spaces developed by Effros, Winkler and Webster. The first substantial duality result for the quantum spaces is a matrix version of the classical bipolar theorem, which asserts that the double matrix (or operator) polar of an absolutely matrix convex and weakly closed matrix set coincides with itself. Actually, this result generalizes Ruan's theorem on a realization of an operator space in the space of all bounded linear operators on a Hilbert space. The matrix bipolar theorem also provides a scale of possible quantizations of a polynormed space, namely the scale of min and max quantizations. Which quantization should play the role of the weak quantum topology associated with a dual pair of linear spaces, and what are weakly matrix bounded sets, that is the main question of our discussion. In the commutative case these problems are solved due to the known classical result by Mackey, and all polynormed topologies compatible with the duality are arranged into the scale within the weak and Mackey topologies thanks to Arens-Mackey theorem. We show that the weak topology of a dual pair admits precisely one quantization and all polynormed topologies compatible with the given duality can be classified by means of a quantum scale on a concrete quantum space. This is a noncommutative Arens-Mackey theorem.