

Banach Algebras 2009
A conference supported by the European Science Foundation under the
ESF-EMS-ERC COM partnership
July 14-24, 2009, Stefan Banach International Mathematical Center, Będlewo,
Poland

Philip Brooker, Mathematical Sciences Institute, Australian National University, Canberra, ACT 0200, Australia; (philip.brooker@anu.edu.au)

Operator ideals on separable $C(K)$ spaces.

ABSTRACT. There are few Banach spaces E for which the lattice of closed, two-sided ideals in $\mathcal{B}(E)$ (the Banach algebra of all bounded linear operators on E) is fully understood. In this talk we give consideration to this problem in the case where $E = C(K)$, the space of continuous scalar-valued functions on a metrisable compact Hausdorff space K (metrisability of K is equivalent to $C(K)$ being norm separable). In particular, we shall discuss classes of operator ideals of natural interest in the study of the closed ideal lattice of $\mathcal{B}(C(K))$.

We first discuss a class $\{\mathcal{D}_\alpha \mid \alpha \in \text{ord}\}$ of operator ideals indexed by the class ORD of ordinals and defined in terms of the *Szlenk index*. The Szlenk index of an operator is an ordinal-valued index that can be thought of as a measure of the ‘Asplundness’ of an operator. After defining the Szlenk index, we introduce the classes \mathcal{D}_α by declaring that an operator T belongs to \mathcal{D}_α if the Szlenk index of T , denoted $\text{Sz}(T)$, does not exceed ω^α .

Time permitting, we shall also discuss a second class of operator ideals, this time indexed by the class of compact Hausdorff spaces. For a given compact Hausdorff space L we denote by \mathcal{H}_L the class of all operators T for which there is $n \in \mathbb{N}$ such that T factors linearly and continuously through a subspace of $C(L) \oplus \cdots \oplus C(L) = C(L)^n$.