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$C(K)$ representations of real Banach algebras

A complex unital Banach algebra can be represented as a space of continuous functions on a compact Hausdorff space via the Gelfand transform. In general it is not possible to represent a real unital Banach algebra as a space of continuous real-valued functions on a compact space without extra conditions.

Among conditions on a real unital Banach algebra we are interested in are the following,

$$\begin{aligned} a^2 + b^2 = e &\text{ implies } \|a^2\| \leq 1, \text{ and} \\ \|a^2\| &\leq k \|a^2 + b^2\|. \end{aligned}$$

We represent the algebra on a subset of its state space. Our main tool is the convexity of the closure of the set of squares in the algebra, an observation derived from the condition under consideration.

A similar results can be obtained via a different path through the maximal ideal space. In contrast with the complex case, an element in a real Banach algebra may have empty spectrum. Take for example the algebra of operators in the plane generated by a rotation and the identity. However, if the algebra under consideration satisfies one of the conditions above, every element has a nonempty spectrum. Also, if the latter condition is satisfied, the Gelfand transform is an isomorphism. The two approaches and connections between them will be discussed, and examples given.