

MATH. 125, QUIZ 9 - Section 5.2 & 5.3 (25points = 5% final grade)

1. (7 points) Find an expression for $\sin 4\alpha$ in terms of $\sin \alpha$ and $\cos \alpha$. Simplify.

Solution:

$$\sin 4\alpha = \sin 2(2\alpha) = 2 \sin 2\alpha \cos 2\alpha = 4 \sin \alpha \cos \alpha (2 \cos^2 \alpha - 1) = 8 \sin \alpha \cos^3 \alpha - 4 \sin \alpha \cos \alpha$$

2. (6 points) Find the exact value of $\tan 22.5^\circ$. Simplify.

Solution:

$$\tan 22.5^\circ = \tan \frac{45^\circ}{2} = \frac{1 - \cos 45^\circ}{\sin 45^\circ} = \frac{1 - \frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}} = \frac{2 - \sqrt{2}}{\sqrt{2}} = \sqrt{2} - 1$$

3. Use the information given about the angle α :

$$\cos \alpha = -0.2, \quad \pi < \alpha < 2\pi$$

to find the exact value of each of the following

- (a) (7 points)

$$\sin 2\alpha = 2 \sin \alpha \cos \alpha = 2 \left(-\sqrt{1 - \left(-\frac{1}{5}\right)^2} \right) \left(-\frac{1}{5} \right) = 2 \cdot \sqrt{\frac{24}{25}} \cdot \frac{1}{5} = \frac{4\sqrt{6}}{25}$$

- (b) (7 points)

$$\sin \frac{\alpha}{2} = \pm \sqrt{\frac{1 - \cos \alpha}{2}} = \sqrt{\frac{1 + \frac{1}{5}}{2}} = \sqrt{\frac{3}{5}}$$