

MATH. 125, QUIZ 8 - Section 5.1 & 5.2 (25points = 5% final grade)

1. (7 points each) Establish the following trigonometric identities; write clearly each step. You may get extra/bonus credit for presenting and explaining your solutions in a professional way (like in the textbook).

(a)

$$\frac{\tan x + \cot x}{\csc x} = \sec x$$

Proof:

$$LHS = \frac{\tan x + \cot x}{\csc x} = \frac{\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}}{\frac{1}{\sin x}} = \frac{\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}}{\frac{1}{\sin x}} \cdot \frac{\sin x \cos x}{\sin x \cos x} = \frac{\sin^2 x + \cos^2 x}{\cos x} = \sec x = RHS$$

(b)

$$1 - \frac{\sin^2 t}{1 + \cos t} = \cos t$$

Proof:

$$LHS = 1 - \frac{\sin^2 t}{1 + \cos t} = \frac{1 + \cos t}{1 + \cos t} - \frac{1 - \cos^2 t}{1 + \cos t} = \frac{\cos t + \cos^2 t}{1 + \cos t} = \frac{\cos t(1 + \cos t)}{1 + \cos t} = \cos t = RHS$$

2. (5 points) NO calculator allowed. Simplify the final answer. Find the exact value of

$$\cos 105^\circ = \cos(45^\circ + 60^\circ) = \cos 45^\circ \cos 60^\circ - \sin 45^\circ \sin 60^\circ = \frac{\sqrt{2}}{2} \cdot \frac{1}{2} - \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} = \frac{\sqrt{2} - \sqrt{6}}{4}$$

3. (8 points) NO calculator allowed. Find the exact value of $\sin 2\alpha$ under the following conditions:

$$\sin \alpha = \frac{1}{3}, \quad \frac{\pi}{2} < \alpha < \frac{3\pi}{2}.$$

Simplify the final answer. You may get extra/bonus credit for presenting and explaining your solutions in a professional way (like in the textbook).

Solution:

$$\sin 2\alpha = \sin(\alpha + \alpha) = \sin \alpha \cos \alpha + \cos \alpha \sin \alpha = 2 \cdot \frac{1}{3} \cdot \left(-\sqrt{1 - \left(\frac{1}{3}\right)^2} \right) = -\frac{2}{3} \sqrt{\frac{8}{9}} = -\frac{2}{3} \cdot \frac{2\sqrt{2}}{3} = -\frac{4\sqrt{2}}{9}$$

Remark: The solution above uses the methods from section 5.2; the same problem can be solved a little faster using 5.3.