Dr. K. Jarosz

# SOLUTIONS

**MATH. 125, QUIZ 16** (25points = 5% final grade)

*Do not use a calculator for problems 1,2, and 3. Show your work. You may use a calculator for problem 4 but you still have to show your work. Write down your answer clearly in an exact form*; partial credit may be given.

1. (7 points) Solve $3\sin^2\theta - \cos^2\theta = 1$, on the interval $0 \le \theta < 2\pi$.
Solution: Since $\cos^2\theta = 1 - \sin^2\theta$ we have
$$3\sin^2\theta - \left(1 - \sin^2\theta\right) = 1$$
so
$$4\sin^2\theta = 2$$
so
$$\sin\theta = \pm\frac{\sqrt{2}}{2}$$
so
$$\theta = \frac{1}{4}\pi, \frac{3}{4}\pi, \frac{5}{4}\pi, \frac{7}{4}\pi$$

2. (7 points) $\sin\alpha = \frac{1}{4}$, $-\frac{\pi}{2} < \alpha < \frac{\pi}{2}$, and $\cos\beta = \frac{1}{3}$, $0 < \beta < \pi$. Find $\sin(\alpha + \beta)$.
Solution:
$$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \sin\beta\cos\alpha = \frac{1}{4}\cdot\frac{1}{3} + \sqrt{1 - \left(\frac{1}{3}\right)^2}\cdot\sqrt{1 - \left(\frac{1}{4}\right)^2} = \frac{1}{12} + \frac{1}{6}\sqrt{30}$$

3. (7 points) Find the exact value of $\tan\left(2\sin^{-1}\left(-\frac{1}{4}\right)\right)$
Solution: Put $x = \sin^{-1}\left(-\frac{1}{4}\right)$ then
$$\sin x = -\frac{1}{4} \text{ and } -\frac{\pi}{2} < x < 0$$
Since
$$\tan^2 x + 1 = \frac{1}{\cos^2 x} = \frac{1}{1 - \sin^2 x} = \frac{1}{1 - \left(-\frac{1}{4}\right)^2} = \frac{16}{15}$$
we get
$$\tan x = -\sqrt{\frac{1}{15}}$$
so
$$\tan\left(2\sin^{-1}\left(-\frac{1}{4}\right)\right) = \frac{2\tan\left(\sin^{-1}\left(-\frac{1}{4}\right)\right)}{1 - \tan^2\left(\sin^{-1}\left(-\frac{1}{4}\right)\right)} = \frac{-2\sqrt{\frac{1}{15}}}{1 - \frac{1}{15}} = -\frac{1}{7}\sqrt{15}$$

4. (3 points each) Consider a triangle with sides 4, 5, 7.
   (a) Find the area of the triangle.
   Answer:
   $$Area = \sqrt{8(8-4)(8-5)(8-7)} \simeq 9.8$$
   (b) Find the angle in degrees of the largest angle in that triangle (round to the closest whole number)
   The largest angle $\alpha$ is located across the longest side and
   $$7^2 = 4^2 + 5^2 - 2\cdot 4\cdot 5\cos\alpha$$
   so
   $$\alpha = \cos^{-1}\left(\frac{4^2 + 5^2 - 7^2}{2\cdot 4\cdot 5}\right) \simeq 102^\circ$$