

MATH. 125, QUIZ 14 - Section 7.3

Show each step of your solution, you do not get credit for just writing the final answer.

1. Find the partial fraction decomposition of each rational expression.

(a) (8 points)

$$\frac{6x + 3}{x(2x - 3)}$$

Solution:

$$\frac{6x + 3}{x(2x - 3)} = \frac{A}{x} + \frac{B}{2x - 3}$$

to find A, B multiply both sides by $x(2x - 3)$

$$6x + 3 = A(2x - 3) + Bx = -3A + (B - 2A)x$$

hence $A = -1; B = 8$ and

$$\frac{6x + 3}{x(2x - 3)} = \frac{8}{2x - 3} - \frac{1}{x}$$

(b) (9 points)

$$\frac{2x - 1}{x^2 + 5x + 6} = \frac{2x - 1}{(x + 3)(x + 2)} = \frac{A}{x + 3} + \frac{B}{x + 2}$$

to find A, B multiply both sides by $(x + 3)(x + 2)$

$$2x - 1 = A(x + 2) + B(x + 3) = (2A + 3B) + (A + B)x$$

hence

$$\begin{aligned} 2A + 3B &= -1 \\ A + B &= 2 \end{aligned}$$

so $A = 7$ and $B = -5$ and

$$\frac{2x - 1}{x^2 + 5x + 6} = \frac{7}{x + 3} - \frac{5}{x + 2}$$

(c) (10 points)

$$\frac{x^2 + x + 2}{(x + 2)^2(x + 3)} = \frac{A}{x + 3} + \frac{B}{x + 2} + \frac{C}{(x + 2)^2}$$

to find A, B, C multiply both sides by $(x + 2)^2(x + 3)$

$$\begin{aligned} x^2 + x + 2 &= A(x + 2)^2 + B(x + 2)(x + 3) + C(x + 3) = A(x^2 + 4x + 4) + B(x^2 + 5x + 6) + Cx + 3C \\ &= (4A + 6B + 3C) + (4A + 5B + C)x + (A + B)x^2 \end{aligned}$$

hence

$$\begin{aligned} 4A + 6B + 3C &= 2 \\ 4A + 5B + C &= 1 \\ A + B &= 1 \end{aligned}$$

so $A = 8, B = -7, C = 4$ and

$$\frac{x^2 + x + 2}{(x + 2)^2(x + 3)} = \frac{4}{(x + 2)^2} - \frac{7}{x + 2} + \frac{8}{x + 3}$$