

MATH. 125, QUIZ 10 - Section 5.5 (25points = 5% final grade)

You are allowed to use a calculator only for the last problem.

1. (9 points each) Solve the following equations on the interval $0 \leq \theta < 2\pi$; if there is no solution explain why not. **Show each step of your solution, you do not get credit for just writing the final answer. You may get extra/bonus credit for presenting and explaining your solutions in a professional way (like in the textbook).**

(a) $\cos 2\theta + 5 \cos \theta + 3 = 0$.

Solution:

Step 1: Replace $\cos 2\theta$ with $2 \cos^2 \theta - 1$ to get after simplification

$$2 \cos^2 \theta + 5 \cos \theta + 2 = 0$$

Step 2: Introduce a name for $\cos \theta = x$ and solve the algebraic equation you obtained:

$$\begin{aligned} 2x^2 + 5x + 2 &= 0 \\ (2x + 1)(x + 2) &= 0, \text{ so} \\ x &= -\frac{1}{2}, -2 \end{aligned}$$

Step 3: Solve $\cos \theta = -2$ and $\cos \theta = -\frac{1}{2}$. The first one does not have any solution. The second one has two solutions (based on the graph of the function): $\theta = \frac{2\pi}{3}$ or $\theta = \frac{4\pi}{3}$

(b) $\sin 2\theta = 2 \sin \theta$.

Solution:

$$\begin{aligned} \sin 2\theta &= 2 \sin \theta \\ 2 \sin \theta \cos \theta &= 2 \sin \theta \\ \sin \theta \cos \theta &= \sin \theta \\ \sin \theta \cos \theta - \sin \theta &= 0 \\ \sin \theta (\cos \theta - 1) &= 0 \\ \sin \theta &= 0 \text{ or } \cos \theta = 1 \\ \theta &= 0, \pi \end{aligned}$$

(c) $\sin^2 x + \sin x = \frac{1}{2}$ (use two decimal points)

Solution:

Step 1: $\sin x = y$:

$$\begin{aligned} y^2 + y - \frac{1}{2} &= 0 \\ y &= \frac{-1 \pm \sqrt{1^2 - 4 \cdot 1 \cdot (-\frac{1}{2})}}{2} \\ y &= \frac{-1 \pm \sqrt{3}}{2} \end{aligned}$$

Step 2: The - sign gives the value of y below -1 so there is no solution for $\sin x = \frac{-1-\sqrt{3}}{2}$. For the other one we get

$$\begin{aligned} \sin x &= \frac{-1 + \sqrt{3}}{2} \\ x &= \sin^{-1} \left(\frac{-1 + \sqrt{3}}{2} \right) \simeq 0.37, \text{ and also} \\ x &= \pi - \sin^{-1} \left(\frac{-1 + \sqrt{3}}{2} \right) \simeq 2.77. \end{aligned}$$