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SOLUTIONS

MATH. 125, QUIZ 10 - Section 5.4 and part of 5.5 (25points = 5% final grade)

Show your work. You may get extra/bonus credit for presenting and explaining your solutions in a professional way (like in the textbook). NO calculator allowed.

1. (5 points) Verify the following trigonometric identity

$$\frac{\sin 2\alpha + \sin 4\alpha}{\cos 2\alpha + \cos 4\alpha} = \tan 3\alpha$$

Proof:

$$LHS = \frac{\sin 2\alpha + \sin 4\alpha}{\cos 2\alpha + \cos 4\alpha} = \frac{2\sin\frac{2\alpha+4\alpha}{2}\cos\frac{2\alpha-4\alpha}{2}}{2\cos\frac{2\alpha+4\alpha}{2}\cos\frac{2\alpha-4\alpha}{2}} = \frac{\sin\frac{2\alpha+4\alpha}{2}}{\cos\frac{2\alpha+4\alpha}{2}} = \frac{\sin 3\alpha}{\cos 3\alpha} = \tan 3\alpha = RHS$$

2. (5 points) Write $\cos(5\alpha)\cos\alpha$ as a sum of cos functions.

Solution:

$$\cos(5\alpha)\cos\alpha = \frac{1}{2}\left[\cos(5\alpha-\alpha)+\cos(5\alpha+\alpha)\right] = \frac{1}{2}\left[\cos(4\alpha)+\cos(6\alpha)\right].$$

3. (5 points) Find exact value of the following expressions; simplify. Hint: using sum-to-product formula or product-to-sum formula will greatly simplify the calculations.

(a)

$$\sin\frac{\pi}{12} + \sin\frac{7\pi}{12}$$

Solution:

$$\sin\frac{\pi}{12}+\sin\frac{7\pi}{12} = 2\sin\frac{\frac{\pi}{12}+\frac{7\pi}{12}}{2}\cos\frac{\frac{\pi}{12}-\frac{7\pi}{12}}{2} = 2\sin\frac{8\pi}{24}\cos\left(-\frac{6\pi}{24}\right) = 2\sin\frac{\pi}{3}\cos\frac{\pi}{4} = 2\cdot\frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2} = \frac{\sqrt{6}}{2}$$

4. Solve the following equations for $0 \le x < 2\pi$

(a) (5 points)

$$\sin x = -\frac{\sqrt{3}}{2}$$

Solution (based on the graph of the function):

$$x = \frac{4\pi}{3}, \frac{5\pi}{3}$$

(b) (7 points)

$$\cos 3x = \frac{1}{2}$$

Solution (based on the graph of the function):

$$\begin{aligned} 3x &= \pm\frac{\pi}{3} + 2k\pi = \frac{\pm\pi + 6k\pi}{3}, \text{ so} \\ x &= \frac{\pi}{9}, \frac{5\pi}{9}, \frac{7\pi}{9}, \frac{11\pi}{9}, \frac{13\pi}{9}, \frac{17\pi}{9}, \end{aligned}$$