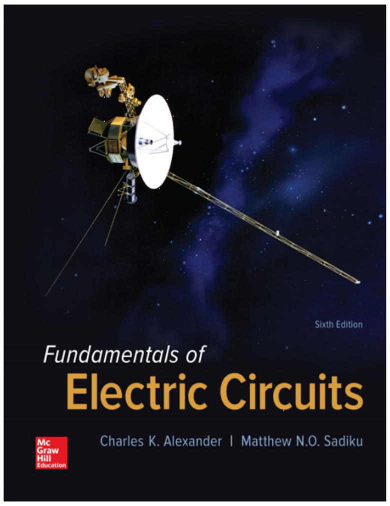


Chapter 3 - Methods of Analysis

- **With Ohm's and Kirchhoff's law established, they may now be applied to circuit analysis.**
- **Two techniques will be presented in this chapter:**
  - **Nodal analysis, which is based on Kichhoff current law (KCL)**
  - **Mesh analysis, which is based on Kichhoff voltage law (KVL)**
- **Any linear circuit can be analyzed using these two techniques.**
- **The analysis will result in a set of simultaneous equations which may be solved by ~~Cramer's rule~~ or computationally (using MATLAB for example)**
- ~~**Computational circuit analysis using PSpice will also be introduced here.**~~

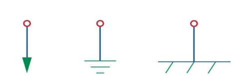
3.1 Introduction

Ohm's law and Kirchhoff's laws can be applied to the analysis of electrical circuits using one of two techniques: **nodal analysis** (which is based Kirchhoff's current law) and **mesh analysis** (which is based on Kirchhoff's voltage law.

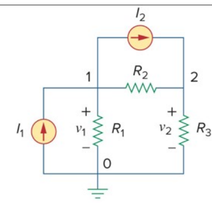
3.2 Nodal Analysis

Nodal analysis focuses on the voltages at the nodes of the circuit rather than the voltages across the circuit elements. This reduces the number of equations that must be solved simultaneously.

Given a circuit that does not contain voltage sources and has *n* nodes, the nodal analysis is accomplished through the application of 3 steps:

- Select a node as the reference node and assign voltages (v_1, v_2, v_3, . . .) to the remaining nodes.
  - The voltages are relative to the reference node.
  - The reference (or datum) node is commonly referred to as the ground since its voltage is, by default, zero.
    - Common symbols for indicating a reference node are: 
- Apply KCL to each of the non-reference nodes. Use Ohm's law to express branch currents in terms of node voltages.
- Simultaneously solve the resulting equations to obtain the unknown node voltages.

Apply nodal analysis to this circuit:

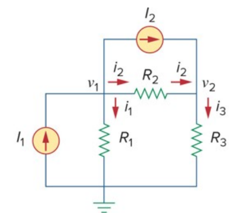


One of the nodes is designated as the ground or node 0.

The remaining two nodes are identified as nodes 1 & 2.

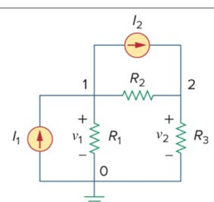
Voltages v_1 and v_2 are assigned to the remaining nodes.

Currents i_1, i_2, and i_3 are shown as the currents through each of the circuit elements (resistors in this case).



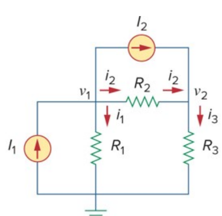
Apply KCL at each node: $I_1 = I_2 + i_1 + i_2$ and $I_2 + i_2 = i_3$

Ch 3, p3



Now use Ohm's law to express the unknown currents in terms of node voltages.

Keep in mind that resistors are passive elements and, by the passive sign convention, **current must always flow through a resistor from a higher potential to a lower potential**.



This leads to:

$$i_1 = \frac{v_1 - 0}{R_1}$$
$$i_2 = \frac{v_1 - v_2}{R_2}$$
$$i_3 = \frac{v_2 - 0}{R_3}$$

Substituting back into the node equations:

$$I_1 = I_2 + \frac{v_1}{R_1} + \frac{v_1 - v_2}{R_2}$$
$$I_2 + \frac{v_1 - v_2}{R_2} = \frac{v_2}{R_3}$$

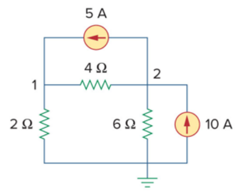
The last step is to solve the system of equations.

3 [n] nodes leads to 2 [n- 1] equations to solve.

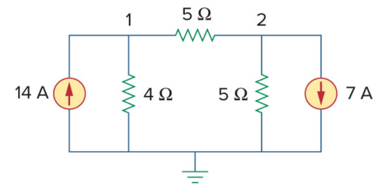
Ch 3, p4

Example and Practice Problems

E3.1 Find the node voltages for this circuit:

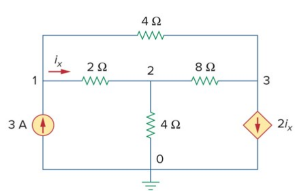


P3.1 Find the node voltages for this circuit:

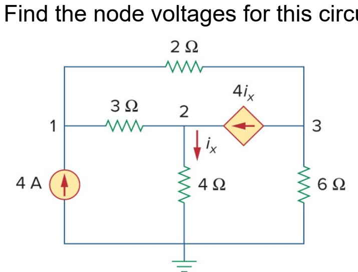


Ch 3, p5

E3.2 Determine the node voltages for this circuit:



P3.2 Find the node voltages for this circuit:

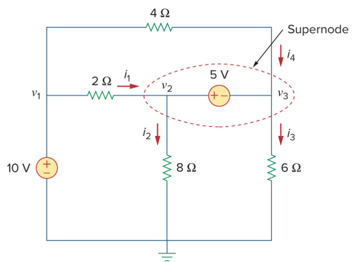


Ch 3, p6

3.2 Nodal Analysis with Voltage Sources

There are two possibilities if the circuit contains a voltage source: (1) the voltage source is connected between the reference node and non-reference node or (2) it is connected between two non-reference nodes.

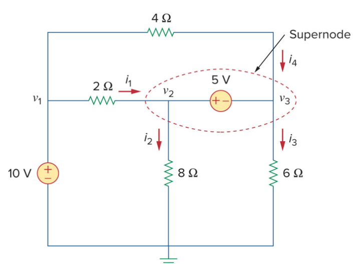
- In the first case, the voltage at the non-reference node is simply set equal to the voltage of the voltage source.
- In the second case, the two non-reference nodes from a generalized or *supernode*.



A ***supernode*** is formed by enclosing a (dependent or independent) voltage source connected between two non-reference nodes and any elements connected in parallel with it.

Note the following properties of a supernode:

- It has no voltage of its own.
- The voltage source inside the supernode provides a constraint equation needed to solve for the node voltages.
- A supernode requires the application of both KCL and KVL to find the node voltages.



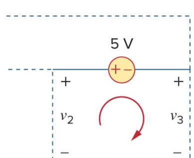
For this supernode:

KCL results in: $i_1 + i_4 = i_2 + i_3$

or

$$\frac{v_1 - v_2}{2} + \frac{v_1 - v_3}{4} = \frac{v_2 - 0}{8} + \frac{v_3 - 0}{6}$$

To apply KVL, redraw part of the circuit as shown and traverse the circuit in a clockwise direction:

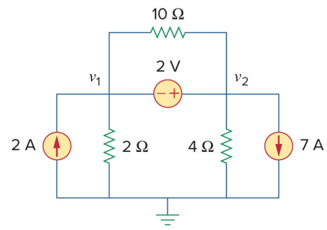


KVL results in: $-v_2 + 5 + v_3 = 0 \Rightarrow v_2 - v_3 = 5$

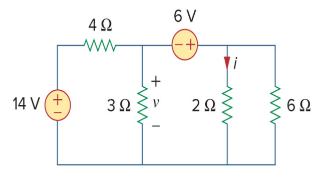
Solving the final two equations simultaneously (using v_1 = 10) results in finding the values for the other node voltages.

Example and Practice Problems

E3.3 Find the node voltages in this circuit.



P3.3 Find v and *i* in this circuit.



3.4 Mesh Analysis

Mesh analysis (also sometimes known as loop analysis) uses mesh currents instead of element currents as the circuit variables, uses KVL to find any unknown current, and has the benefit of reducing the number of equations that must be solved simultaneously.

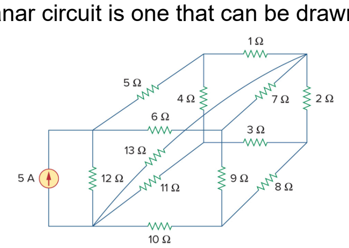
Nodal analysis applies KCL to find unknown voltages in a given circuit, while mesh analysis applies KVL to find unknown currents.

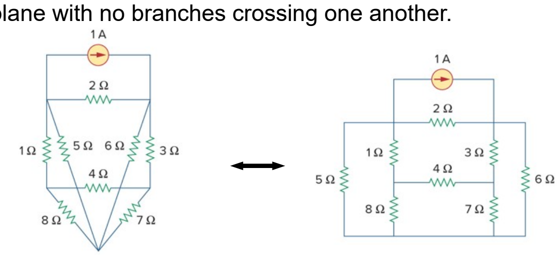
Remember: A loop is a closed path with no node being passed more than once.

A mesh is a loop that does not contain any other loops within it.

Mesh analysis is limited in one important respect and can only be applied to circuits that are planar.

A planar circuit is one that can be drawn in a plane with no branches crossing one another.

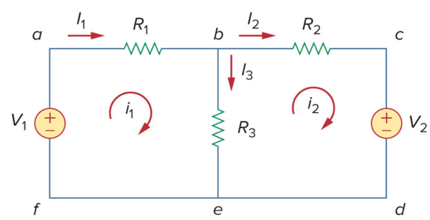




Given a circuit that does not contain current sources and has *n* meshes, mesh analysis is accomplished through the application of 3 steps:

- Assign currents (i_1, i_2, i_3, . . .) to the n meshes.
- Apply KVL to each of the *n* meshes. Use Ohm's law to express the voltages in terms of the mesh currents.
- Simultaneously solve the resulting *n* equations to obtain the mesh currents.

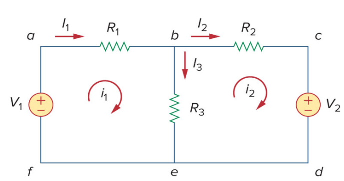
Apply mesh analysis to this circuit.



Some things to note:

The direction assigned to the mesh current is arbitrary, but the convention is to assume a clockwise direction for the flow of each mesh current.

The branch currents are different from the mesh currents. Branch currents are indicated by upper-case I while the mesh current is indicted by lower-case i.



Mesh currents i_1 and i_2 are assigned to meshes 1 and 2.

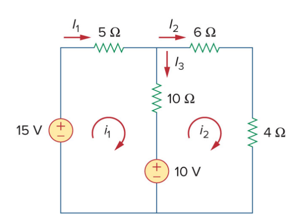
Apply KVL to each mesh and noting that branch current $I_3 = i_1 - i_2$:

$$-V_1 + R_1 i_1 + R_3(i_1 - i_2) = 0 \quad \Downarrow \quad (R_1 + R_3)i_1 - R_3 i_2 = V_1$$

$$R_2 i_2 + V_2 + R_3(i_2 - i_1) = 0 \quad \Downarrow \quad -R_3 i_1 + (R_2 + R_3)i_2 = -V_2$$

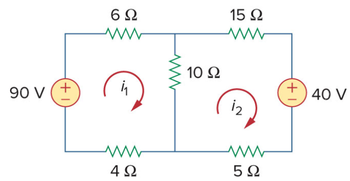
If a circuit has *n* nodes, *b* branches, and l independent loops or meshes, then *l* = *b* - *n* + 1 and *l* independent,simultaneous equations are required to solve the circuit using mesh analysis.

E 3.5 Find the branch currents of this circuit using mesh analysis.

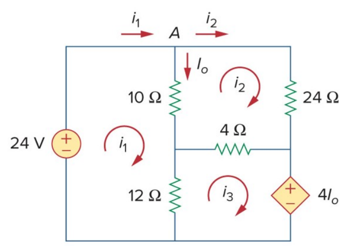


Example and Practice Problems

P3.5 Calculate the mesh currents of the circuit shown below:



E3.6 Develop the equations needed to find current I_0 in this circuit.

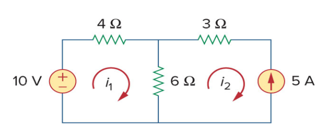


3.5 Mesh Analysis with Current Sources

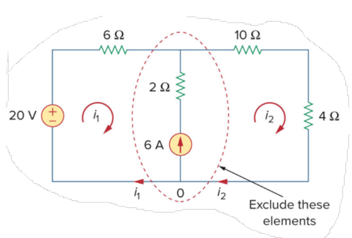
Applying mesh analysis to circuits that contain current sources (dependent or independent) actually simplifies the analysis since the number of equations is reduced.

There are two possibilities if the circuit contains a current source: (1) the current source exists only in one mesh or (2) exists between two meshes.

- In the first case, the mesh current is simply set equal to the current from the current source.



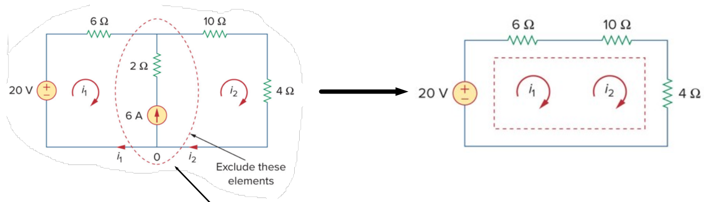
- In the second case, a supermesh is created by excluding the current source and any circuit elements connected in series with it.



A ***supermesh*** results when two meshes have a (dependent or independent) current source in common.

Note the following properties of a supermesh:

- It has no current of its own.
- The current source in the supermesh provides the constraint equation needed to solve for the mesh currents.
- A supermesh requires the application of both KCL and KVL.



For this supermesh:

Applying KVL to the suppermesh results in:

$$-20 + 6i_1 + 10i_2 + 4i_2 = 0 \qquad \text{or} \qquad 6i_1 + 14i_2 = 20$$

Applying KCL to the node in the branch where the two meshes intersect:

$$i_2 = i_1 + 6$$

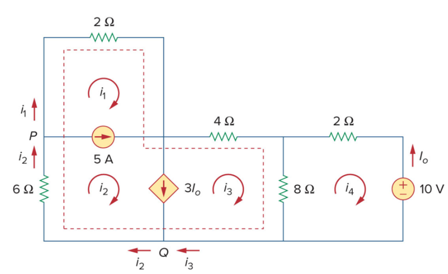
Solving these two equations simultaneously allows you to find values for both mesh currents:

i_1 = -3.2 A $\qquad$ i_2 = 2.8 A

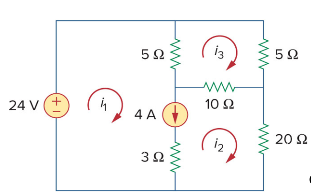
Ch 3, p15

Example and Practice Problems

E3.7 Use mesh analysis to develop the equations for all 4 mesh currents.



P3.7 Use mesh analysis to develop the equations for all 3 mesh currents in the following figure.



Change the 4A source to 12A to match what the author actually did and then solve for the mesh currents.

Ch 3, p16