Chapter 6

Problems, Distinctions, and Interpretations

6.1 Puzzles

Quantum mechanics was born in controversy, in the midst of a conflict of ideas and, apparently, of egos. Heisenberg's abstract matrix mechanics provided no picture of the quantum world and was roundly criticized by Schrödinger, who at first thought that quantum particles could be identified with the waves associated with TDSE. Bohr, Heisenberg, and Pauli, on the other side, resisted Schrödinger's attempt to restore an intuitively satisfying picture of the electron as a very small wave packet.[1] They turned out to be right. Indeed, after an initial and misleading success in constructing well behaved (non-spreading) wave packets from the superposition of eigenfunctions of quantum harmonic oscillators in stationary states, Schrödinger himself recognized that typically wave packets spread, and in a system made of more than one particle, the wave propagates in configuration space, not in ordinary space, where presumably atoms, the components of our physical world, exist.

Schrödinger was also convinced that he could do away with the discontinuity of quantum jumps associated with collapse by using superpositions of eigenfunctions for stationary states. Here too he was wrong. At the end, his version of quantum mechanics, although often mathematically handier, proved no more intuitively satisfying than

[1] For a time, Born was rather sympathetic to Schrödinger's view. However, he ended up by interpreting the wave function statistically and not as representing a real wave, and Schrödinger himself, by the end of 1926, had adopted Born's statistical interpretation. For a detailed account of these early debates, see Beller, M., (1999).

Heisenberg's. Einstein repeatedly expressed his theoretical dissatisfaction with the new theory in lectures, conferences, and in his correspondence, arguing with Bohr in a series of thought experiments that have become part of the lore of 20th century physics.[2]

The fact is that quantum mechanics is puzzling for several reasons. For starts, not only are its results often very disconcerting (tunneling!), but its mathematical apparatus looks also strangely removed from physical reality. Even granting the danger of trying to read off a physical theory's interpretation from its formalism, Newton's equations seem to codify the behaviors of physical objects in terms of position, velocity, and force, all features of the physical world we can directly experience. By contrast, TDSE, with its ineliminable imaginary components and its existence in abstract space, seems to defy any natural interpretation. Indeed TDSE may appear closer to the common view of Ptolemy's abstruse epicycles and equants than to Kepler's laws. As Ptolemy's circles might be viewed just as computational devices predicting where planets are in the heavens, so TDSE seems just a computational device capable of delivering Ψ, the square modulus of which allows one to obtain the probability distributions that account for the apparent waviness of quantum particles. Does quantum mechanics give a true representation of the quantum world or is it 'just' a predictive device?

In addition, quantum mechanics seems to introduce a bizarre degree of randomness in the behavior of quantum particles, which appear to act indeterministically. One is left wondering whether randomness is a real feature of the quantum world or just an appearance caused by our ignorance. Could the apparent indeterministic behavior of quantum particles be just the unfortunate outcome of the theory's incompleteness in the

[2] For a brief discussion, see Greenstein, G., and Zajonc, A., (1997): 85-92.

sense that, contrary to the claims of standard quantum mechanics, state vectors provide only incomplete (if correct) information about quantum systems? Could quantum mechanics be modified and interpreted in such a way as to eliminate randomness, perhaps by adding some extra information (the so-called "hidden variables") to that contained in state vectors? Or should we look for a deeper and completely new theory in the light of which indeterminacy will disappear, as Einstein thought?

Another disconcerting feature of standard quantum mechanics has to do with measurement and the alleged attendant collapse of the system's state. What happens exactly to bring about such a dramatic change in the temporal evolution of the system? After all, the interaction between the quantum system under study and the measuring apparatus just consists in an energy exchange between the two, the sort of physical process TDSE should deal with. Why then, at measurement, the state vector, and therefore the physical state the vector represents, is not governed by TDSE but by the collapse postulate? In other words, why does TDSE's job merely seem temporally to propel a packet of potential experimental returns while the experiment, with the attendant collapse postulate, seems to bring about the emergence of one specific experimental result?

All of these issues are both complex and interrelated, and will occupy us from now on. In the process, we shall learn more quantum mechanics and the philosophical concepts we shall employ in our investigation.

6.2 Realism

Most of us believe that physical objects, their physical states, and their properties, exist independently of us, of our language, of our conceptual schemes, and of our practices; in other words, most of us are realists with respect to physical objects and their features. One can go further and adopt scientific realism, roughly the view that well established scientific theories provide us with (reasonably) observer-independent information about the physical world, and that therefore they give us a (reasonably) true representation of it.[3] However, for our purposes, in agreement with our commonsensical attitudes, let us assume that a realist is someone who adopts the following two related views.

First, physical objects exist independently of our minds. For example, the Moon really exists on its own independently of whether we experience it or even of whether we exist or not: it is really there even when nobody is looking, to quip on Einstein's remark (Pais, A., (1979): 863). The same is true of atoms, electrons, or other quantum particles we can manipulate or on which we can perform direct experiments.[4]

Second, the characteristics of such physical objects are definite, mind-independent, and measurement independent. Here, of course, one may be discriminating,

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[3] Scientific realism has been challenged. For example, one could adopt instrumentalism, the view that scientific theories are merely instruments for the correct prediction of phenomena. Indeed, one may also challenge realism altogether, and hold that the physical world is mind-dependent.

[4] Here we need not address the issue whether "theoretical" entities exist. Most our discussions will revolve, or can be made to revolve, around atoms and electrons, and one can make the case that they have ceased being theoretical entities a long time ago. For example, we have been able to push individual atoms around in prefixed patterns for quite some time, as the famous 1990 image of the IBM logo made of xenon atoms shows.

depending on which physical characteristics one picks. For example, one might hold that at all times an electron is in a definite physical state (which may or may not be completely represented by the state vector), that is, be a realist with respect to quantum states, and perhaps quantum states only, a view adopted by Einstein. Let us call this view "physical state realism." Or, one might hold that an electron has physical properties with definite values independently of minds or measurements. This last view can be taken in (at least) two ways. One may claim that an object has all its physical properties all the times and hold, for example, that an electron has definite, mind-independent, and measurement independent position and momentum all the times. Let us call this view "strong property realism." Alternatively, one may hold that at any time quantum particles have only quantum mechanically compatible definite, mind-independent, and measurement independent physical properties. For example, electrons do not have both *x*-spin component and *z*-component spin at the same time but can have, say, *z*-component spin and position at the same time. Let us call this view "weak property realism." [5]

At this point one might claim that although the fact that most of us are realists about tables, chairs, stars, and what not militates in favor of property realism, whether one is a realist is strictly a matter of philosophical preference about which physics has nothing to say. As we shall see, this is not quite so if we are prepared to make property realism more than a purely metaphysical position by connecting it to experience. More specifically, if we want quantum mechanics to have anything to say about property

[5] Note that state realism and property realism of either sort are not mutually exclusive. For example, one might hold that at all times an electron is in a definite quantum state and has all of its properties.

realism, we need to say more about such properties. Hence, we stipulate that property realism assumes that *the values of properties are eigenvalues* of the relevant operators, that is, that they are the measurement returns. In other words, we build this requirement into our view of property realism. The assumption is reasonable. We know that the results of measurement are eigenvalues; all we need to add is the Faithful Measurement Principle, namely, that the (competent) measurement of O reveals the value of O.

6.3 Determinism

Often one hears that standard quantum mechanics is an indeterministic theory. It is useful to distinguish three related but distinct types of determinism: evolutionary determinism, computational determinism, and value determinism. In contrast to realism, which, in our understanding of the term, applies primarily to the world, these notions apply primarily to the theory one considers, and only secondarily (if at all) to the physical world.

As the name implies, evolutionary determinism applies to the temporal evolution of a physical system as described in a physical theory. Typically, the temporal development of the system is ruled by an equation which, provided the initial conditions for time t_0, has one and only one (particular) solution for any other time t_n. Since TDSE is first-order with respect to time, it satisfies evolutionary determinism. In addition, if the equation is time reversible, as TDSE is, then t_n can be earlier or later than t_0 and evolutionary determinism extends to the past as well as to the future. Hence, as far as evolutionary determinism is concerned, quantum mechanics is not different from classical mechanics.[6]

Computational determinism has to do with the fact that it is one thing to know that given certain initial conditions a differential equation has one and only one solution, but it is another to be able to compute it. As it turns out, TDSE is typically fiendishly difficult to solve, with the result that analytical (that is, textbook nice) solutions are few, and most of the times one has to resort to numerical approaches nowadays using computers. In addition, the system may be very sensitive to its previous states, so that even small errors in the determination of the initial conditions drastically limit one's ability to predict the state of the system even moderately ahead. This is especially evident in chaotic systems, described by non-linear equations and very sensitive to initial conditions, which are evolutionarily deterministic but often not computationally deterministic. Hence, while computational determinism entails evolutionary determinism, the converse is not true. Of course, whether we can solve an equation or not may have important practical or even theoretical consequences, but conceptually, as far as the issue of determinism is concerned, it is of little consequence.

Value determinism (a better name might be "value definiteness" or perhaps "value sharpness") applies when the measurements of the same observable on any two systems, which are in the same physical state *according to the theory*, always produce the same

[6] Although this may be a bit of an oversimplification, Newton's equation of motion, $m\frac{d^2}{dt^2}x = F$, also results in evolutionary determinism.

result.[7] Value determinism obtains in classical mechanics; for example, the classical mechanical state of a material particle fully determines the particle's position and momentum, and therefore all other observables. However, value determinism does not obtain in standard quantum mechanics: two systems in a quantum state represented by the very same state vector can give different returns when the same observable is measured. Value determinism, however, applies to static properties. For example, an electron always has the same mass, the same electrical charge -1, and the same spin number 1/2, no matter its quantum state. To be an electron is, among other things, to have such immutable characteristics. Similarly, a proton always has mass 1836 (a mass 1836 times bigger than that of the electron), electrical charge +1, and spin 1/2. By contrast, observables such as position, momentum, energy, or spin orientation fail to satisfy value definiteness. Value determinism is not entailed either by computational or evolutionary determinism, as any of the systems for which analytical solutions were provided shows.

The issues surrounding value determinism are, of course, associated with the state vector and its probabilistic interpretation, and loomed large in early controversies on the completeness of quantum mechanics. One can easily see why. For example, a strong property realist will conclude that standard quantum mechanics must be incomplete because it rejects value determinism and therefore can only provide probabilistic predictions. In other words, she will argue, there must be more to the physical state of the particle than the state vector tells us.

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[7] In this case, the observable is said to be *sharp*. When the condition is not satisfied (according to standard quantum mechanics, the system is in a state of superposition), the observable is said to be *fuzzy*.

6.4 The Values of Observables

As we saw, the issue of value determinism is one of the obvious features separating standard quantum mechanics from classical mechanics. Let us consider it in some detail. As we know, the measurement of a static (specific) property *M* such as mass always returns the same results. This fact is unproblematic, and if all observables exhibited this characteristic, as far as value determinism is concerned standard quantum mechanics would be on a par with classical mechanics. For one could maintain, as in classical mechanics, that just before the measurement the particle had property *M* and that the measurement revealed its magnitude to us. However, when it comes to dynamic (individual) observables, the situation gets murky. For, one must distinguish two cases.

If the system is in an eigenstate $|\Psi_i\rangle$ of $\hat{N}$, then *N*'s measurement will certainly return the appropriate eigenvalue λ_i. If, by contrast, we consider an observable *O*, and the system is not in an eigenstate of $\hat{O}$, then the measurement will have one among a spread of possible returns. *N*'s case, at least if we as usual restrict ourselves to ideal measurement, is similar to that exhibited by static observables. Since every time we measure *N* we get λ_i, it seems reasonable to assume that just before the measurement the particle had property *N* and that the measurement revealed its magnitude. However, the case with respect to *O* is radically different because all that we obtain is a spread of probabilities. It is important to understand what this means according to standard quantum theory, and to do this, let us look at the difference between pure and mixed cases.

6.5 Pure and Mixed States

Up to now, we have always assumed that the state vector $|\Psi\rangle$ is fully known. To be sure, since $|\Psi\rangle = \{c_1|e_1\rangle + ... + c_n|e_n\rangle\}$, the state vector may be a complicated linear combination of the basis vectors, but even so, we have assumed that all the components and their proportion in the mixture making up $|\Psi\rangle$ are known. Then, the system is in a pure state. However, pure states are the exception rather than the rule. Very often, the information we have about the state of the system is incomplete, and we have to make do with a statistical mixture of vectors. All we know is that the system has probability $p_1, p_2, \ldots, p_n$ of being in the corresponding state $|\Psi_1\rangle, |\Psi_2\rangle, \ldots, |\Psi_n\rangle$. Of course, $p_i \geq 0$ and $\sum_i p_i = 1$. The system is then in a (proper) mixed state. [8] Notice that saying that a system is in a mixed state refers to the fact that we do not know the exact composition of the system state, although we know it is definitely in one of the states represented by a $|\Psi_i\rangle$. An epistemological claim of this sort is not peculiar to quantum mechanics, since often we have only partial knowledge of physical systems, be they quantum or classical.

Let *O* be an observable, $\hat{O}$ the operator representing it and $\hat{O}|\Xi_i\rangle = \lambda_i|\Xi_i\rangle$. Suppose now that we want to determine the probability of obtaining λ_i upon measuring *O* in the mixed state system just described. Since if the system is in state $|\Psi_1\rangle$ then

[8] Here we deal only with proper mixed states. There are also improper mixed states, which we shall consider later. As usual, we assume that all the state vectors have been normalized. However, $|\Psi_1\rangle, |\Psi_2\rangle, \ldots, |\Psi_n\rangle$ need not be orthogonal.

$\Pr(\lambda_i) = |\langle \Xi_i | \Psi_1 \rangle|$, and similarly for the other states, the probability of obtaining λ_i in the case of the mixed state will simply be

$$\Pr(\lambda_i) = \sum_i p_i |\langle \Xi_i | \Psi_i \rangle|^2 .^9 \tag{6.5.1}$$

One must be careful not to confuse a linear superposition with a statistical mixture. Suppose that $|\Psi\rangle$ is a linear superposition of $|\Psi_1\rangle$ and $|\Psi_2\rangle$:

$$|\Psi\rangle = c_1|\Psi_1\rangle + c_2|\Psi_2\rangle. \tag{6.5.2}$$

Now by $\Pr_{|\Psi\rangle}(\lambda)$ let us denote the probability of obtaining λ as a measurement return if the system is in state $|\Psi\rangle$. Then,

$$\Pr_{|\Psi\rangle}(\lambda_n) = |\langle \Xi_n | \Psi \rangle|^2, \tag{6.5.3}$$

$$\Pr_{|\Psi_1\rangle}(\lambda_n) = |\langle \Xi_n | \Psi_1 \rangle|^2, \tag{6.5.4}$$

$$\Pr_{|\Psi_2\rangle}(\lambda_n) = |\langle \Xi_n | \Psi_2 \rangle|^2. \tag{6.5.5}$$

By plugging (6.5.2) into (6.5.3), we obtain

$$\Pr_{|\Psi\rangle}(\lambda_n) = \left|\langle \Xi_n | \left(c_1|\Psi_1\rangle + c_2|\Psi_2\rangle\right)\rangle\right|^2 = |c_1\langle \Xi_n | \Psi_1 \rangle + c_2\langle \Xi_n | \Psi_2 \rangle|^2. \tag{6.5.6}$$

Developing the square, we have

$$\Pr_{|\Psi\rangle}(\lambda_n) = |c_1|^2|\langle \Xi_n | \Psi_1 \rangle|^2 + |c_2|^2|\langle \Xi_n | \Psi_2 \rangle|^2 + 2c_1c_2^*\langle \Xi_n | \Psi_1 \rangle\langle \Xi_n | \Psi_2 \rangle^*. \tag{6.5.7}$$

By plugging in (6.5.3) and (6.5.4),

$$\Pr_{|\Psi\rangle}(\lambda_n) = |c_1|^2 P_{|\Psi_1\rangle}(\lambda_n) + |c_2|^2 P_{|\Psi_2\rangle}(\lambda_n) + 2c_1c_2^*\langle \Xi_n | \Psi_1 \rangle\langle \Xi_n | \Psi_1 \rangle^* . \tag{6.5.8}$$

However, if we read (6.5.2) not as a linear superposition, but as a statistical mixture of $|\Psi_1\rangle$, and $|\Psi_2\rangle$ with probability weights $|c_1|^2$ and $|c_2|^2$, then

[9] Hence, the rule for expectation values is $<O>= \sum_i p_i \langle \Psi_i | \hat{O} | \Psi_i \rangle$.

$$P_{|\Psi\rangle}(\lambda_n) = |c_1|^2 P_{|\Psi_1\rangle}(\lambda_n) + |c_2|^2 P_{|\Psi_2\rangle}(\lambda_n), \quad (6.5.9)$$

which is incompatible with (6.5.2) because, among other things, it leaves out $2c_1c_2^*\langle\Xi_n|\Psi_1\rangle\langle\Xi_n|\Psi_2\rangle^*$, the mathematical representation of the interference between wave functions $|\Psi_1\rangle$ and $|\Psi_2\rangle$.

Since linear superpositions are not statistical mixtures, we may not understand (6.5.2) as saying that the system is definitely in state $|\Psi_1\rangle$ (in which case the eigenstate to eigenvalue link will give us $O = \lambda_1$) or definitely in state $|\Psi_2\rangle$ (in which case we shall have $O = \lambda_2$). In other words, there is no straightforward way of interpreting superpositions in terms of ignorance.

6.6 Pre-Measurement Observables

Since linear superpositions cannot be understood as (proper) mixed states, we are left with an obvious question: what should we say about the existence and value of an observable *O* just before the measurement if its predicted return value is not sharp? Three positions come to mind:

1. *O* existed and had a definite value.
2. *O* did not exist.
3. The question is meaningless, or al least physically irrelevant, because it has no verifiable answer.

The first two positions provide different answers, while the third rejects the question on philosophical grounds. Let us look at them briefly.

According to the first position, effectively amounting to property realism, *O* existed and had a definite eigenvalue that measurement revealed. The fact that in standard quantum mechanical predictions *in principle* (not merely because of measurement limitations) we have to make do with probabilities and expected values is just a manifestation of theoretical weakness.[10] Since standard quantum mechanics is unable to tell us *O*'s value, and instead gives us a spread of probabilities, the theory is incomplete, and the most plausible explanation for this is that, contrary to the orthodox interpretation, the state vector cannot be the whole story when it comes to describing the quantum state of a particle.[11]

Ideally, then, we should come up with a new theory T involving some so called *hidden variables*, collectively denoted by the symbol λ, which together with $|\Psi\rangle$ provide

[10] The qualification is crucial. For standard quantum mechanics, knowing all that there is to be known about its quantum state (that is, its state vector) does not eliminate probabilistic predictions: quantum jumps are an ineliminable part of the theory.

[11] It is possible to adopt a sort of property contextualism to the effect that measurement returns (the relevant eigenvalues) depend not only on the system under scrutiny but also on the measurement settings (environmental contextualism) or on the observable that are measured at the same time (algebraic contextualism). However, whether such a position can be reasonably considered property realism is doubtful. If measurement returns depend on the context and measurement are faithful, it is hard to see how what is being measured is a true property of the system under study. If we take it as bona fide property realism, property contextualism can also be strong or weak. Note that if one adopts it, one should still try to construct a hidden variable theory S with the same general features of T but with one difference: λ would involve variables related not only to the system under study but to the context as well.

a complete description of quantum states. At least in principle, $|\Psi\rangle$ plus λ would be able fully to determine the result of any individual measurement. Of course, T would have to agree with quantum mechanics wherever the latter agrees with experience. In particular, T would have to generate some account of why we get the expectation values we get, and why we cannot prepare ensembles of systems in which two incompatible observables provide the same returns when measured. This would require the use of a suitable probability distribution on λ which would generate the appropriate expectation values.[12]

Different hidden variable theories have been put forth, the De Broglie-Bohm theory being the most developed one. It reproduces the results of standard quantum mechanics, and we shall discuss it later.

6.7 The Orthodox Interpretation

According to the she second position, *O* did not exist. It is the most widely held view, and a direct result of EE (the eigenstate-eigenvalue link), which the orthodox interpretation adopts. Since $|\Psi\rangle = \sum_n c_n |\psi_n\rangle$ is a linear superposition, not a statistical mixture, before measurement the system's state was not one of the $|\psi_n\rangle$'s. But since *O* can return λ_n only if $|\Psi\rangle = |\psi_n\rangle$, *O* had no value at all and therefore did not exist. The

[12] Broadly speaking T would work this way. Given a system in some state $|\Psi\rangle$, an observable *O* would have a sharp value $[O_\Psi]$ which would be a function of λ; in other words, it would be the case that $[O_\Psi](\lambda)$. In addition, λ could not be directly known or controlled, and for a state $|\Psi\rangle$, it could take different values. Now let $\delta_\Psi(\lambda)$ be the probability density of λ with respect to the state $|\Psi\rangle$. Then, *O*'s expectation value for a system in state $|\Psi\rangle$ would be $\langle O_\Psi\rangle = \int [O_\Psi](\lambda)\delta_\Psi(\lambda)d\lambda$.

system acquires a value for O only through measurement, when $|\Psi\rangle$ collapses either by interacting with the measuring apparatus or because of the intervention of a conscious observer. As a result, observables exist only insofar as their values are sharp (and then the system is in the relevant eigenstate) or are actually measured (in which case, because of collapse, EE takes over).

In addition, since $|\Psi\rangle$ can be expanded into a set of eigenvectors of any observable, it is the measuring apparatus (and ultimately the observer) that determines what sort of property the system will actually exhibit, although, of course, the individual or the average returns are not up to the observer. Still, as Jordan is said to have colorfully claimed, "[O]bservations not only disturb what has to be measured, they produce it. In a measurement of position, the electron is forced to a decision. We compel it to assume a definite position; previously it was neither here nor there, it had not yet made its decision for a definite position." (Jammer, M., (1974): 161). Some, for example Pauli and Heisenberg, claimed that we may understand the properties in-between measurements to exist as potentialities that are brought to actuality by the interaction of the quantum particles with the measurement apparatus, but this is not a significant deviation from the orthodox view.

6.8 The Last View

The third position, which we may dub "the verificationist interpretation", holds that the question whether O existed and had a value is meaningless, because no empirical evidence could support or undermine it. The rationale behind this view is provided by empiricism, the position that all knowledge and all concepts used to obtain it are based on the experience we obtain from the senses. Under a radical form of empiricism, not only all the concepts that are not appropriately reducible to sense experience but also all statements that cannot be tested in experience must be rejected. [13] Since all that is physically accessible to us is measurement returns, asking whether *O* existed and had a value is nonsensical.

If one does not like such a radical form of empiricism, one might retreat to an apparently more defensible view, namely that although the question whether *O* existed and had value is meaningful, nevertheless it is physically inane, and as such more in the realm of metaphysics than physics. However, as we shall see, some results show that under some reasonable (although by no means absolutely compelling) assumptions it does make an empirical difference whether before the measurement *O* existed and had a definite value, and therefore a statement like "*O* had a definite value" is neither meaningless nor physically inane.

[13] Of course, empiricists disagree on what "appropriately reducible" and "tested in experience" really involve, but the idea should be clear enough for our purposes.

Exercises

Exercise 6.1

1. A spin-half particle is in state $|\Psi\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow_x\rangle + |\downarrow_x\rangle\right)$. What shall we get if we measure S_z?

2. Consider $W = \frac{1}{2}\left(|\uparrow_x\rangle + |\downarrow_x\rangle\right)$, a mixed state such that the particle is determinately in state $|\uparrow_x\rangle$ with probability 1/2 or in state $|\downarrow_x\rangle$ with probability 1/2. What shall we get if we measure S_z?

Exercise 6.2

1. True or false: if the value of an observable is zero then the system does not have that property.

2. True or false: in the orthodox interpretation, the kind of dynamical observables a system possesses depends on the experimenter.

Answers to the Exercises

Exercise 6.1

1. Since $|\uparrow_z\rangle = \frac{1}{\sqrt{2}}(|\uparrow_x\rangle + |\downarrow_x\rangle)$, we shall get $\hbar/2$ with probability 1.

2. Since $|\uparrow_x\rangle = \frac{1}{\sqrt{2}}(|\uparrow_z\rangle + |\downarrow_z\rangle)$, if the particle is in state $|\uparrow_x\rangle$ we shall get $S_z = \hbar/2$ with probability 1/2 or $S_z = -\hbar/2$ with probability 1/2. Similarly, since $|\downarrow_x\rangle = \frac{1}{\sqrt{2}}(|\uparrow_z\rangle - |\downarrow_z\rangle)$, if the particle is in state $|\downarrow_x\rangle$ we shall get $S_z = -\hbar/2$ with probability 1/2 or $S_z = \hbar/2$ with probability 1/2. Hence, the overall probability of obtaining $S_z = \hbar/2$ is 1/2.

Exercise 6.2

1. False. To have value zero, a property must exist. For example, a particle at rest has velocity with value zero.

2. True. Measuring *O* causes the state vector to collapse onto an eigenvector of $\hat{O}$, and EE guarantees that S has property *O* with a definite value. Had we decided to measure *Q* instead, S would now have property *Q* with a definite value. In addition, if $\hat{O}$ and $\hat{Q}$ do not share eigenvectors, EE implies that if S has property *O* it does not have property *Q*.