Chapter 13

Non-Local Strong Property Realism (of a Kind): Bohmian Mechanics

Bell's theorem allows strong property realism as long as it is non-local, and KS allows it as long as enough properties have a sufficient degree of contextuality to prevent the application of formulas like (9.6.1) and (9.6.2). In this chapter, we consider Bohm's mechanics (BM), a theory that adopts a peculiar variety of non-local property realism in which quantum particles have only the property of position while other 'properties', such as spin, are contextual and, strictly speaking, no true properties at all.

13.1 Bohm's Mechanics

In 1952, David Bohm produced the first workable theory that attributes deterministic trajectories to quantum particles. The physical intuition behind the attempt was not new, as it had already been proposed by de Broglie in 1927. It consisted in turning Schrödinger's wave into a real wave that physically affects the particle associated with it by guiding the particle's motion. There are different versions of BM; here we start by staying rather close to Bohm's own original formulation. Since we need to manipulate Ψ, a complex function, before we start we must learn how to solve complex equations.

A complex number can always be written in the form $x+iy$, and by definition, two complex numbers are equal if and only if both their real and their imaginary parts are equal. For example, $x+iy=3+2i$ if and only if $x=3$ and $y=2$.[1] Hence, to solve a complex equation, we separate the real and the imaginary components, we set them equal

[1] Somewhat misleadingly, the imaginary part of $x+iy$ is *y*, not *yi*. The mathematical treatment in 13.1 can be merely perused on a first reading.

to zero, and then we solve the two real numbers equations. For example, to solve the equation above, we write $x-3=0$ and $y-2=0$. Then we solve to obtain $x=3$ and $y=2$. We can now address Bohm's theory.

The first postulate of the theory is that, as in standard quantum theory, TDSE is the evolution equation of Ψ. Let us start by rewriting Ψ in terms of two real functions (functions containing only real numers), $S(x,t)$ and $R(x,t)$, so that

$$\Psi = R\cos\left(\frac{S}{\hbar}\right) + iR\sin\left(\frac{S}{\hbar}\right) = Re^{\frac{iS}{\hbar}}. \tag{13.1.1}$$

R is then the positive amplitude of the wave and *S* its phase function.

We can now rewrite TDSE as

$$i\hbar\frac{\partial}{\partial t}\left(Re^{\frac{iS}{\hbar}}\right) = -\frac{\hbar^2}{2m}\frac{\partial}{\partial x^2}^2\left(Re^{\frac{iS}{\hbar}}\right) + VRe^{\frac{iS}{\hbar}}.^2 \tag{13.1.2}$$

After a simple and short calculation, the left side of (13.1.2) becomes

$$i\hbar\frac{\partial R}{\partial t}e^{\frac{iS}{\hbar}} - R\frac{\partial S}{\partial t}e^{\frac{iS}{\hbar}}. \tag{13.1.3}$$

Similarly, the right side of (13.1.2) becomes

$$-\frac{\hbar^2}{2m}e^{\frac{iS}{\hbar}}\left[\frac{\partial^2 R}{\partial x^2} + \frac{2i}{\hbar}\frac{\partial R}{\partial x} + \frac{i}{\hbar}R\frac{\partial^2 S}{\partial x^2} - \frac{R}{\hbar^2}\left(\frac{\partial S}{\partial x}\right)\right]^2 + VRe^{\frac{iS}{\hbar}}. \tag{13.1.4}$$

Putting (13.1.3) and (13.1.4) together, eliminating the common exponential, and separating the real and the imaginary terms, one gets

$$i\hbar\left[\frac{\partial R}{\partial t} + \frac{1}{m}\frac{\partial R}{\partial x}\frac{\partial S}{\partial x} + \frac{R}{2m}\frac{\partial^2 S}{\partial x^2}\right] = R\frac{\partial S}{\partial t} - \frac{h^2}{2m}\frac{\partial^2 R}{\partial x^2} + \frac{R}{2m}\left(\frac{\partial S}{\partial x}\right)^2 + VR. \tag{13.1.5}$$

This is a complex equation, and the solution is provided by the two real equations

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[2] For an introduction to derivatives, see appendix one.

$$\frac{\partial R}{\partial t} = -\frac{1}{2m}\left(R\frac{\partial^2 S}{\partial x^2} + 2\frac{\partial R}{\partial x}\frac{\partial S}{\partial x}\right) \tag{13.1.6}$$

and

$$\frac{\partial S}{\partial t} = -\left[\frac{1}{2m}\left(\frac{\partial S}{\partial x}\right)^2 + V - \frac{h^2}{2m}\frac{\partial^2 R}{\partial x^2 R}\right]. \tag{13.1.7}$$

Now let us add a second postulate by setting

$$v = \frac{1}{m}\frac{\partial S}{\partial x}, \tag{13.1.8}$$

which gives the "guidance condition' for the particle's trajectory by determining its velocity. [3]

We are now in a position to see how the theory works. TDSE determines Ψ up to a constant factor, and Ψ gives us *R* and *S*. [4] Note also that (13.1.1) entails that $R = |\Psi|$. All that is required to have a deterministic particle trajectory in ordinary space is the particle's initial position, which, in principle, can be discovered experimentally.[5] In

[3] In other words, the particle's momentum (*mv*) is the gradient of the phase function of the wave. Consequently, it is perpendicular to the lines of constant phase of the wave function. Note that the particle's velocity depends both on the particle's position and on the state of the associated pilot wave.

[4] *S*, the phase function of Ψ, is not uniquely determined since *S* and $S' = S + 2\pi\hbar$ produce the very same Ψ; however, as $\frac{\partial}{\partial x}S = \frac{\partial}{\partial x}(S + 2\pi\hbar) = p$, $\frac{\partial S}{\partial x}$ is single valued, and therefore this lack of uniqueness has no physical consequence.

[5] Remember, however, that as TDSE is linear, any superposition of Ψ's is also a solution. Hence, in contrast to classical mechanics, the same classical potential *V* is

short, given the initial conditions $\Psi_0(x)$ and x_0 (this is the so-called "hidden variable"), the particle's trajectory is uniquely determined for all times.

However, in practice, background noise and instrumental imprecision make it impossible to determine the initial conditions of quantum particles exactly. Hence, we need to use an initial probability density function, and BM postulates (this is the third postulate) that once normalized $|\Psi_0|^2 = (R_0)^2$ is the initial position density function at time t_0.[6] This is similar to the orthodox postulate, but not identical to it. For, while in the orthodox interpretation $|\Psi_0|^2 = (R_0)^2$ provides information only about position measurement returns, in BM it provides information about the actual positions of the particles. Moreover, the third postulate is necessary only because of our ignorance: as for classical statistical mechanics, could we determine the exact positions of individual particles, we could dispense with probability.

13.2 The Guidance Condition

To get a sense of how the guidance condition works, let us look at the (standard) notion of probability density flow. Suppose we shoot a particle in the right direction towards an area *a*-*b*, where a detector can register the presence of particles (Fig. 1).

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compatible with an infinity of particle trajectories even when the initial position and velocity are specified.

[6] Actually, Bohm has argued that even if the initial distribution does not satisfy the third postulate, random interactions among the particles would lead to the postulated distribution $|\Psi_0|^2 = (R_0)^2$. In short, for Bohm, the third postulate is not strictly necessary.

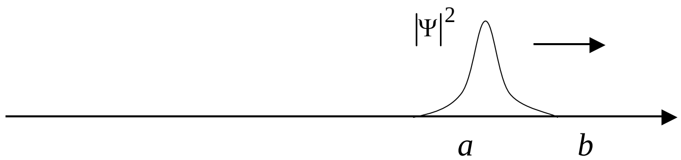

Figure 1

As the particle moves to the right (that is, as the probability of detecting the particle increases as we move to the right), the probability $\Pr(a,b)$ of detection in the region *a-b* is given by the area under $|\Psi|^2$ and between *a* and *b*. So, if $|\Psi|^2$ has not yet reached *a* $\Pr(a,b)$ is zero; as $|\Psi|^2$ passes *a* initially $\Pr(a,b)$ increases, reaching a maximum when $|\Psi|^2$ is in the middle of the segment *a-b*, and then it decreases, eventually becoming zero again as the whole of $|\Psi|^2$ moves to the right of *b*. One could say that the probability density of detection flows into *a-b* through *a* and out of *a-b* through *b*, *as if* it were a fixed quantity (normalization!) of probability density fluid. The analogy can be carried further by introducing the idea of a probability density current moving to the right, where the current is defined by the amount of fluid constituting it and by the velocity with which it moves.

Given how in BM the particle's velocity *v* is defined by (13.1.8), the probability density current $j(x,t)$ at *x* at time *t* turns out to be

$$j(x,t) = R^2 v. \tag{13.2.1}$$

In other words, at any time and position, the particle's velocity is proportional to, and has the same direction as, the probability density current at those very same time and position. So, the trajectory of the particle is exactly the line along which probability density flows: it is as if the particle were carried along by the probability density current. The outcome is that if at time t_0 a particle's (epistemic) probability of being in the region *c*-*d* is given by the area under $|\Psi_0|^2$ and between *c* and *d*, at a later time t_i the (epistemic) probability of the particle being in the region *e*-*f* is given by the area under $|\Psi_i|^2$ and between *e* and *f*, just as quantum mechanics predicts. In general, it can be shown that BM agrees with the statistical returns predicted by quantum mechanics.

13.3 Some Basic Features of the Theory

BM clearly rejects the state completeness principle since the specification of the wave function of a system does not afford all the information about the system: as we saw, one must add the particle's position. BM also rejects EE, as the state function need not be a Dirac delta function in order for the particle to be at a determined position. In addition, contrary to the standard interpretation, a particle has both determinate position and momentum at the same time.

Since the non-classical features of the theory flow from the wave function, let us look at it in some detail. Ψ represents an objectively existing wave, so that BM's basic ontology consists of particles and pilot waves which have, however, peculiar properties. A pilot wave does not propagate in ordinary space but in configuration space of dimension 3*n*, where *n* is the number of the particles in the system. Consequently, as the wave is taken to be real, configuration space must be real as well (that is, just as real as ordinary space). In addition, contrary to classical physics, while the pilot wave dynamically affects the particle the converse is not true. In other words, there is no reciprocity of action between the particle and the wave. The Ψ-field has the further bizarre characteristic of not having any apparent source in that there is no localized entity generating it. For example, the Earth's gravitational field is produced by the mass of our planet, a highly localized object; nothing similar occurs in the quantum case, even if Ψ does depend on the particle's mass.

13.4 The Harmonic Oscillator

Let us see how BM treats the quantum harmonic oscillator. However, before we do that, it may be helpful to make two general points about trajectories. First, whatever trajectory a particle may have, it cannot pass though the nodes of Ψ's plot, where $\Psi = 0$. The reason is that there $\frac{\partial S}{\partial x}$ is undefined because S could have any value. This is as it should be, since quantum mechanics tells us that the probability of finding a particle at the nodes is zero. Second, where $\frac{\partial S}{\partial x}$ is defined it is single valued, and consequently given a time *t* only one trajectory passes through a given point. It follows that trajectories have the peculiar property of not touching or crossing each other.

The generic pilot wave for a stationary state of the quantum harmonic oscillator is given by

$$\psi_n(x,t) = h_n(x)e^{\frac{-iE_n t}{\hbar}} \quad , \tag{13.4.1}$$

where *n* is a positive integer, $h_n(x)$ is proportional to the appropriate Hermite polynomial, and, as we saw, the system's energy is

$$E_n = \left(\frac{1}{2} + n\right)\hbar\omega \text{ .[7]} \tag{13.4.2}$$

[7] Hermite polynomials are well known mathematical beasts that need not concern us, although we should note that in (13.4.1) the polynomial depends on the energy level *n*.

As

$$p=\frac{\partial S}{\partial x}=\frac{\partial}{\partial x}\left(\frac{-iE_n t}{\hbar}\right)=0, \qquad (13.4.3)$$

the particle is actually at rest![8] By hypothesis, the position distribution is the same as in standard quantum mechanics, as it is given by $|\Psi_n|^2$.

13.5 Properties and Measurement

According to BM, particles have only one intrinsic dynamical property, position. Other quantum 'properties' are all contextual, some quite radically so, as we can see by considering spin. To anticipate a bit, according to BM, particles do not have spin; rather, to put it crudely, like everything else that is strange, spin is associated with the pilot wave. Mathematically, this is done by modifying (13.1.8) to allow spinors. However, we need not go into that. Rather, let us look at a 'spin measurement' according to BM. The key idea here is that BM makes use of the standard state vector whose evolution is linear because it is ruled by TDSE.

Suppose (Fig. 2) that we shoot an electron, initially at position *a*, through a SGZ and that the associated wave is represented by

$$|\Psi\rangle=\psi_a(x)\otimes|\uparrow_z\rangle, \qquad (13.5.1)$$

where $\psi_a(x)$ refers to a wave that is not zero at point *a*.

[8] To get the particle to move, we need to form superposed states that in Bohm's mechanics, albeit not in classical mechanics, make the system non-conservative. For a great wealth of examples together with a systematic exposition of Bohm's theory, see Holland, P., (1993).

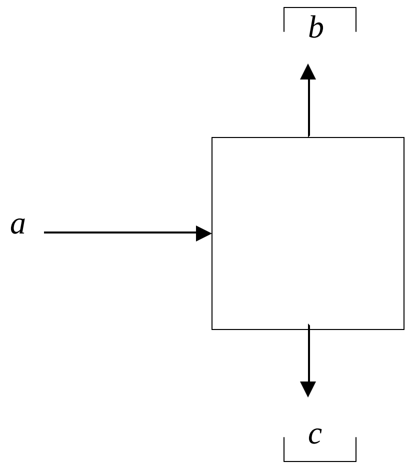

Figure 2

(Note that $\psi_a(x)$ need not be a Dirac delta function centered at *a*). As the particle plus wave enter the SGZ, the inhomogeneous magnetic field of the device will deflect the wave upwards with the result that the particle will arrive at *b*, and its final wave function will be

$$|\Psi\rangle = \psi_b(x) \otimes |\uparrow_z\rangle. \tag{13.5.2}$$

This process is completely deterministic: given (13.5.1) and the magnetic field of the SGD, the electron could only get at *b*. Had the initial wave been

$$|\Psi\rangle = \psi_a(x) \otimes |\downarrow_z\rangle, \tag{13.5.3}$$

the electron would have arrived at *c* and its final wave function would have been $|\Psi\rangle = \psi_c(x) \otimes |\downarrow_z\rangle$. If we place detectors at *b* and *c*, an orthodox theorist will calibrate our device so that a click at *b* is associated with $S_z = \hbar/2$ and one at *c* with $S_z = -\hbar/2$.

Since the wave evolution is linear, if the initial wave is represented by

$$|\Psi\rangle = \alpha\psi_a(x) \otimes |\uparrow_z\rangle + \beta\psi_a(x) \otimes |\downarrow_z\rangle, \tag{13.5.4}$$

the final wave will be represented by

$$\left|\Psi'\right\rangle = \alpha\psi_b(x)\otimes\left|\uparrow_z\right\rangle + \beta\psi_c(x)\otimes\left|\downarrow_z\right\rangle. \quad (13.5.5)$$

If the action of the SGZ on the wave is sufficiently strong (that is, if all other influences can be neglected), the wave represented by (13.5.5) will be effectively split into two non-overlapping branches, an upward one ($\psi_b(x)\otimes\left|\uparrow_z\right\rangle$) and a downward one ($\psi_c(x)\otimes\left|\downarrow_z\right\rangle$). To make things easy for us, let us assume a perfectly symmetrical state of affairs so that $|\alpha|^2 = |\beta|^2 = 1/2$; then, the upper half of the original wave Ψ will be deflected upwards and the lower half downwards. Hence, if *a*, the original position of the electron, is in the upper half of Ψ, then the electron will arrive at *b*, the detector will click and the orthodox physicist will claim that the electron has now *z*-spin-up ($S_z = \hbar/2$); by contrast, if *a* is in the lower half of Ψ, the particle will arrive at *c* and the orthodox physicist will claim that the electron has now *z*-spin-down ($S_z = -\hbar/2$). The statistical outcome will be exactly that of quantum mechanics: half the electrons will produce a click in the upper detector and the rest a click in the lower detector, and the orthodox physicist will interpret the result accordingly.[9]

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[9] Here we have followed Albert, D. Z., (1992): 146-48. Note that there is no collapse in BM. Since the branches of the final wave do not overlap, the particle is in one and only one of the wave packets, and, moreover, the guidance condition when evaluated at the particle's location, depend only on the wave packet in which the particle resides. Consequently, the other 'empty' wave packets can be *computationally* discarded, the Bohmian counterpart of orthodox collapse. Empty wave packets are as real as non-empty ones. However, in spite of the fact that they propagate energy though space, they cannot be detected because it turns out that they interact only with other empty packets. This is

However, a Bohmian physicist will interpret the result differently, claiming that we have explained a 'spin measurement' merely in terms of the electron's motion, without attributing spin to it. Insisting that the clicking at the detector signifies that the electron's *z*-spin component has a certain value is not required; rather, it is the natural outcome of a naively realist attitude towards quantum mechanical operators, of the view that each of them represents a true property of the particle. At this point, one might reply that although the measurement result does not require one to assume that the electron has spin, still this is the most reasonable interpretation of the experiment. However, in BM such a response can be met by considering the degree of contextuality that characterizes spin.

As we noted, for BM the only absolutely intrinsic (dynamical) property of a particle is position for, as Bohm has noted, even velocity depends on the wave function. In other words, in a single particle system, the only dynamical property ever definable independently of the wave function is the initial position. So, one might consider, as Bohm does, all other dynamical properties as contextual. Still, we should remember that for BM the quantum state of a particle is described by both position and wave function, and the particle's quantum state completely determines its velocity. So, the contextuality of velocity simply depends on the fact that the quantum state of a particle is itself contextual in that it does not depend on the particle alone but requires the pilot wave as well. However, the contextuality of spin is much more radical, for in certain situations the outcome of a 'measurement of spin' is not determined even if the initial quantum

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unfortunate because their discovery would provide obvious support to Bohm's theory (Holland, P., (1993): 371-73).

state of the particle (particle's position plus wave) is given, a strange outcome, given the deterministic structure of BM.

Imagine a perfectly symmetrical state of affair in which the initial state of an electron is

$$|\Psi\rangle = \frac{1}{\sqrt{2}}\psi_a(x)\otimes|\uparrow_z\rangle + \frac{1}{\sqrt{2}}\psi_a(x)\otimes|\downarrow_z\rangle, \qquad (13.5.6)$$

and suppose we send it through a SGZ.[10] Then, if position *a* is in the upper half of the wave, then the electron will end up at position *b* and will display henceforth a *z*-spin-up behavior. For example, if we send it again through a SGZ, it will invariably emerge from the *z*-spin-up exit. By contrast, if *a* is in the lower half of the initial wave, then the electron will end up at *c* and henceforth display *z*-spin down behavior. Suppose, however, that while the electron is traveling towards the SGZ, we reverse the SGZ's polarity. If *a* is in the upper half of the wave, then the electron will, as before, end up at *b*; however, it will display *z*-spin-*down* behavior. Similarly, if *a* is in the lower half of the initial wave, then the electron will end up at *c*; however, it will display *z*-spin-*up* behavior. In other words, in spite of BM being a deterministic theory, the initial quantum state of the electron is not sufficient to determine the spin behavior of the particle. To do that, one must also specify the state of the measuring device. Spin is contextual in the radical sense that its determination requires not only the initial specification of both the particle's position and the wave (which together give a complete description of the initial physical state of the particle) but also that of the instrument, which has nothing to do with the initial state of the particle. It is this last feature of spin that lends plausibility to the idea that spin is not a true property of the particle. So, while in standard quantum

[10] Here we follow Albert, D. Z., (1992): 153-55.

mechanics all the observables are on the same footing both mathematically (since the eigenvectors of Hermitian operators span the Hilbert space) and physically (standard quantum theory is no more about position than, say, spin), in Bohm's theory only the mathematical equivalence is preserved. Position and velocity are real properties in that they are both obtainable from the particle's quantum state, but spin, for example, is not. The radical contextuality of all dynamical properties not directly reducible to position prevents their attribution prior to, or independently of, measurement, thus blocking the reasoning leading to KS (Bohm, D., and Hiley, B. J., (1988): 118-20).

13.6 The Quantum Potential

As BM reproduces the results of quantum mechanics, one could stop here, as it were. Indeed, some proponents of BM do just that. However, others, including Bohm himself, have gone further by introducing a new entity, the quantum potential. Up to now, Schrödinger's wave has been made real, and we have been told that it determines the motion of the particle, but the dependence of the latter on the former seems more mathematical than physical. Looking for a physical story, it seems reasonable to assume that the changes in the particle's velocity requires some force which can explain how the wave affects the particle.

Let us start by taking the derivative of (13.1.8) with respect to time to obtain

$$\frac{dp}{dt}=\frac{d}{dt}\left(\frac{\partial S}{\partial x}\right)=v\frac{\partial}{\partial x}\left(\frac{\partial S}{\partial x}\right)+\frac{\partial}{\partial t}\left(\frac{\partial S}{\partial x}\right)=\frac{p}{m}\frac{\partial}{\partial x}\left(\frac{\partial S}{\partial x}\right)+\frac{\partial}{\partial t}\left(\frac{\partial S}{\partial x}\right).^{11} \qquad (13.6.1)$$

By utilizing (13.1.8) again, we get

[11] Here we are employing the rule $\frac{d}{dt}g=\frac{\partial}{\partial t}x\frac{\partial}{\partial x}f+\frac{\partial}{\partial t}f$, where $g=g(x,t)$ and $f=x(t)$.

$$\frac{dp}{dt}=\frac{1}{m}\frac{\partial S}{\partial x}\frac{\partial}{\partial x}\left(\frac{\partial S}{\partial x}\right)+\frac{\partial}{\partial t}\left(\frac{\partial S}{\partial x}\right), \tag{13.6.2}$$

and by the product rule for derivatives in reverse we have

$$\frac{dp}{dt}=\frac{\partial}{\partial x}\left[\frac{1}{2m}\left(\frac{\partial S}{\partial x}\right)^2+\frac{\partial S}{\partial t}\right]. \tag{13.6.3}$$

Finally, by plugging (13.1.7) into (13.6.3), simplifying, and using

$$Q=-\frac{h^2}{2m}\frac{\partial^2 R}{R\partial x^2} \tag{13.6.4}$$

to clean up the notation we obtain

$$\frac{d}{dt}p=-\frac{\partial}{\partial x}(V+Q), \tag{13.6.5}$$

Bohm's Equation of Motion. It is similar to Newton's Equation of Motion as long as one treats $V+Q$ as the system's potential energy. This amounts to introducing a new quantity Q, the *quantum potential energy*, in addition to the classical V. Consequently, as the first summand in the right side of (13.1.7) is the particle's kinetic energy, one may reasonably set

$$\frac{\partial S}{\partial t}=-E, \tag{13.6.6}$$

where E is the system's energy.[12]

It may be worth considering how BM can use the quantum potential in the quantum harmonic oscillator we considered before. Remembering that

$$E=\left[\frac{1}{2m}\left(\frac{\partial S}{\partial x}\right)^2+V+Q\right], \tag{13.6.7}$$

that

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[12] Since momentum is $p=mv$ and kinetic energy is $K=\frac{1}{2}mv^2$, $K=\frac{p^2}{2m}$.

$$V(x) = \frac{1}{2} m\omega^2 x^2, \tag{13.6.8}$$

and that the particle is at rest, we obtain

$$Q = \left(\frac{1}{2} + n\right)\hbar\omega - \frac{1}{2} m\omega^2 x^2. \tag{13.6.9}$$

Since (13.6.5) shows that when $Q = 0$ particles behave exactly as classical ones, the quantum potential is responsible for quantum effects and BM can provide a very precise theoretical account of when a system turns classical. In practice, when $\frac{Q}{V} << 1$ the results of Bohmian mechanics are effectively indistinguishable from those of classical mechanics.[13] Physically, the most interesting characteristic of Q is its dependence on the wave function. Because of (13.6.4), Q is proportional to the second position derivative of $|\Psi|$, and therefore it is very sensitive to $|\Psi|$'s shape; moreover, it is insensitive to $|\Psi|$'s magnitude. Consequently, even where $|\Psi|$ is close to zero, as long as there are sudden sharp spikes and bumps, albeit small, its influence on Q, and therefore on the particle's trajectory, can be profound and need not diminish with distance.

In spite of its explanatory role, the quantum potential remains a mysterious entity. It is essentially related to Ψ as the means through which the wave influences the particle. The weird thing here is that the wave does not classically push the particle because its effects on the particle are not proportional to the square of its magnitude, and therefore are not proportional to its intensity. This is especially clear in situations where Q is very

[13] However, one should not conclude that BM is classical mechanics plus the quantum potential, as the two theories differ with respect to what determines the velocity of the particle. In classical mechanics it is the force alone; in BM it is also the position.

small in comparison to the particle's kinetic energy. As Bohm has noted, a particle's trajectory can be guided by Q much as the route of a ship on automatic pilot is guided by radar waves carrying energy that is negligible with respect to that present in the ship. But of course, this is just an analogy, not an explanation. Moreover, the physical basis for the quantum potential is shrouded in mystery. At worst, Q still looks just like a mathematical artifact, the inevitable result of (13.1.2) and (13.1.8), without any physical foundation.

13.7 Uncertainty and Non-Locality

For BM, a particle has both position and momentum all the times, and therefore all the versions of the uncertainty principle are interpreted in terms of ignorance. In addition, since the initial particle distribution in the wave is given by $|\Psi|^2$, and the derivation of GUP (and HUP) holds in BM, the statistical outcomes of BM will satisfy GUP. More specifically, imagine that we have a narrow wave packet (a narrow pilot wave) of width δ_x. If we measure momentum, the wave packet will branch into a set of wave packets of width $1/\delta_p$, where δ_p is the width of a branch in momentum space. Hence, if the accuracy of the momentum measurement is great (δ_p is small), then the original wave packet has spread a great deal and the particle could be just about anywhere with equal probability. With respect to HUPI, Bohm has adopted a kind of disturbance view, arguing that any measurement changes the shape of the wave function. For example, in a typical measurement, as the wave branches, the effective wave (that containing the particle) is more localized than the original one (Bohm, D., and Hiley, B. J., (1993): 110). So, for example, if we measure momentum we alter the pilot wave, thus giving the particle a new momentum different from the initial one.

One of the most remarkable characteristics of BM is its overt adoption of non-locality as it is evident once we consider entangled systems, where a particle's velocity (and therefore its trajectory) is instantaneously affected through Ψ by the positions of the other particles. We can see this even more clearly in an EPR situation in which the wave function is

$$|\Psi\rangle = \frac{1}{\sqrt{2}}\left(\psi_a^1(x) \otimes |\uparrow_z^1\rangle \otimes \psi_d^2(x) \otimes |\downarrow_z^2\rangle\right) - \frac{1}{\sqrt{2}}\left(\psi_a^1(x) \otimes |\downarrow_z^1\rangle \otimes \psi_d^2(x) \otimes |\uparrow_z^2\rangle\right) \tag{13.7.1}$$

Suppose that particle 1 is in the upper half of its wave and enters a SGZ. Then, its effective wave will land it at *b* and the particle will exhibit *z*-spin-up behavior. In the meantime, the wave will have evolved to

$$|\Psi'\rangle = \frac{1}{\sqrt{2}}\left(\psi_b^1(x) \otimes |\uparrow_z^1\rangle \otimes \psi_d^2(x) \otimes |\downarrow_z^2\rangle\right) - \frac{1}{\sqrt{2}}\left(\psi_c^1(x) \otimes |\downarrow_z^1\rangle \otimes \psi_d^2(x) \otimes |\uparrow_z^2\rangle\right) \tag{13.7.2}$$

Since the value of $\psi_c^1(x)$ at *b* is zero, the second term of (13.7.2) will vanish (the wave branch it represents will become an empty wave), so that we are left with the first term, which tells us that particle 2 at *d* will exhibit *z*-spin-down behavior. In particular, even if particle 2 is in the upper half of its wave, it will still emerge a SGZ from the lower path. In short, what has happened to particle 1 has instantaneous consequences on the trajectory of particle 2, a clear example of non-locality. Note that if we could know the initial position of particle 1, a feat possible in principle, we could use the EPR correlation to send superluminal signals. Of course, all of this is purely theoretical. In reality, we cannot know the precise location of particle 1 as our knowledge of it is restricted to the distribution given by $|\Psi|^2$, just as in standard quantum mechanics, and therefore we are unable to send superluminal signals.

Although BM is non-local, since the propagation associated with the quantum potential is mediated through the wave field, Bohm has claimed that it not action at a distance. However, it is instantaneous, and for this reason likely conflict with Special Relativity, a view Bohm has denied by arguing that the non-local features of the quantum potential cannot (in practice) be used to send superluminal signals. As for Einstein's claim that if the world were non-local, physics would be impossible because no system could be sufficiently isolated to allow us to study it, Bohm's answer has been that at the macroscopic level, the only level at which we can actually carry out investigations, non-local effects are insignificant. More generally, he has tended to view principled objection to non-locality as manifestation of mere prejudice.[14]

13.8 Reactions to Bohm's Theory

Although the pilot wave has some distressing characteristics, the very construction of Bohmian mechanics is noteworthy because historically causal theories have not fared well. From the very inception of wave mechanics in 1926, some tried to provide a realistic interpretation of the quantum algorithm. Initially, Schrödinger himself thought of Ψ as representing a real wave encompassing the particle, without much success beyond one-particle systems. In the same year, Erwin Madelung proposed an interpretation of quantum mechanics based on a manipulation of TDSE similar to Bohm's, suggesting that TDSE represents a non-viscous fluid of identical particles of

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[14] See, for example, Bohm, D., and Hiley, B. J., (1993): 157-58. For a discussion of this and other problems, see Callender, C., and Weingard, R., (1997).

mass m, of density $\delta = |\Psi|^2$, and of velocity field $v = \frac{1}{m}\frac{\partial S}{\partial x}$. In the following year, De Broglie advanced a theory mathematically similar to Madelung's in which the wave is real and "guides" the particle through the formula $v = \frac{1}{m}\frac{\partial S}{\partial x}$, as in Bohm's theory.[15] Both theories ran into difficulties. In particular, De Broglie failed to answer objections by Pauli at the fifth Solvay conference in 1927, and he soon became convinced that the whole enterprise was wrongheaded. The death toll for all such attempts was provided, or so it seemed, in 1932 by von Neumann's celebrated theorem that no "hidden variable" theory can produce all the same predictions of quantum mechanics. However, as we saw, the proof is flawed.

Early reactions to Bohm's theory were almost uniformly negative. Einstein did not like the intrinsic non-locality of the quantum potential, and more generally claimed that no mere reinterpretation of the quantum algorithm would be sufficient to set things on the right track. Bohm's approach, he told Born, "seems too cheap to me" (Letter to

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[15] However, while Bohm's theory explicitly postulates the existence of both particle and wave as two distinct and irreducible realities, De Broglie's was more ambiguous. His preferred view was to consider the particle as a mathematical singularity at the center of the wave, thus effectively absorbing the particle into the wave. However, he was also prepared to entertain the idea that the particle has an independent and distinct reality from the wave's, as in Bohm's view. Einstein as well proposed a theory in which the wave determines not a statistical spread but the actual motion of an individual system; however, he soon became convinced that it would not work and never published the manuscript. For details on all these attempts, see Cushing, J. T., (1994): ch. 8.

Born of May 12, 1952, in Born, M., (ed.) (1971): 192). What was needed was a completely new theory. Heisenberg could not see how a wave propagating in configuration space could be real. He also disliked the privileged place that position has in the theory and compared it unfavorably with the conceptual equality enjoyed by all the observables in the standard formalism. More generally, he viewed Bohm's theory as both unnecessary and dangerous: unecessary because it is empirically equivalent to standard quantum theory; dangerous because it toys with metaphysics by postulating the existence of trajectories that are effectively unobservable beyond the results of the orthodox interpretation. The comments by other orthodox theorists like Pauli and Rosenfeld were similarly negative. Only de Broglie, after a critical reaction, changed his mind.

As time went by, Bohm's theory was simply ignored by most physicists. However, some were duly impressed by Bohm's achievement. For example, in 1982 Bell recounted how von Neumann's proof (whose conclusions he knew through Born's account) had disabused him of determinism and convinced him that the statistical character of quantum descriptions is in principle unavoidable. However, he tells us,

> in 1952 I saw the impossible done. It was in papers by David Bohm…. Moreover, the essential idea was one that had been advanced already by de Broglie in 1927, in his 'pilot wave' picture.
>
> But why then had Born not told me of this 'pilot wave'? If only to point out what was wrong with it? Why did von Neumann not consider it? More extraordinarily, why did people go on producing 'impossibility' proofs after 1952, and as recently as 1978? When even Pauli, Rosenfeld and Heisenberg

> could produce no more devastating criticisms of Bohm's version that to brand it as 'metaphysical' and 'ideological'? Why is the pilot wave ignored in textbooks? Should it not be taught, not as the only way, but as an antidote to the prevailing complacency? To show that vagueness, subjectivity, and indeterminism, are not forced on us by experimental facts, but by deliberate theoretical choice? (Bell, J. S., (1982 in Bell (1987): 160).

Although Bell did not try to answer these questions, Cushing has proposed a partial answer. Following a lead by Bohm, he has argued that the hegemony of the orthodox interpretation was the result of historical contingency, not of objectively rational choice, going as far as to present a counterfactual, but in his view plausible, scenario in which in the mid 1920's a realist interpretation of quantum theory wins the day (Cushing, J. T., (1994): ch. 10).

Exercises

Exercise 13.1

1. Show that that Q is insensitive to $|\Psi|$'s magnitude. [Hint. Show that if we multiply $|\Psi|$ by a constant the value of (13.6.4) does not change]
2. True or false: V totally determines Q by determining TDSE.

Exercise 13.2

1. Show that in a stationary state, the effective potential $V + Q$ is independent of time.
2. True or false: At the ground level of the harmonic oscillator, all the energy is given by the quantum potential.
3. True or false: In the harmonic oscillator, the quantum force $-\frac{\partial Q}{\partial x}$ cancels the classical force related to V.

Answers to the Exercises

Exercise 13.1

1. $-\frac{\hbar^2}{2m}\frac{\partial^2(aR)}{aR\partial x^2} = -\frac{h^2}{2m}\frac{\partial^2 R}{R\partial x^2} = Q$.

2. False. Any linear superposition of solutions is a solution of TDSE. Consequently, TDSE does not uniquely determine *R*.

Exercise 13.2

1. *V* is time independent because a stationary state is conservative; *Q* is time independent as well, as (13.6.4) clearly shows.

2. True, since the particle has no kinetic energy.

3. True. $F_Q = -\frac{\partial Q}{\partial x} = m\omega^2 x$, and $F_V = -\frac{\partial V}{\partial x} = -m\omega^2 x$.